PHILOSOPHICAL TRANSACTIONS.

I. *The Solar and Lunar Diurnal Variations of Terrestrial Magnetism.*

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§ 1. *Introduction.*

THE regular daily changes of the earth's magnetism, considered as a world-wide phenomenon, afford a problem of much interest and importance. It is one, moreover, which the researches of BALFOUR STEWART and SCHUSTER have shown to be more vulnerable to attack than seem most of the problems of terrestrial magnetism. But in spite of their success, and of the contributions of subsequent writers, the comprehensive study of the subject has suffered undeserved neglect. It seems an unfortunate fact that the efforts of magneticians are unduly devoted to the accumulation of data, the time and labour spent in their discussion being, proportionately, inconsiderable.

The promising theory that the daily magnetic variations arise mainly from electric currents circulating in the upper atmosphere, under the impulsion of electromotive forces produced by the convective motion of the air across the earth's permanent magnetic field, was first propounded by BALFOUR STEWART.* In a simple but penetrating discussion of the features both of the solar and lunar diurnal variations, he showed the power of this theory to account for the facts in a way which none of the other theories then current could do.

The theory was greatly developed, and rendered more definite, in two important memoirs by SCHUSTER in 1889 and 1907.† Adopting a suggestion by GAUSS, he applied the method of spherical harmonic analysis to the solar diurnal magnetic variations, to determine whether they had their origin mainly above or below the

* *Cf.* his article on "Terrestrial Magnetism" in the 9th edition of the 'Encyclopædia Britannica' (1882). He made considerable use of BROUN'S admirable study of the lunar diurnal variation of declination at Trevandrum (1874).

† SCHUSTER, 'Phil. Trans.,' A, vol. 180, p. 467 (1889); and A, vol. 208, p. 163 (1907).

earth's surface. The result demonstrated the accuracy of BALFOUR STEWART'S conclusion that the origin must be external.

At the close of his first paper SCHUSTER suggested that the convective atmospheric motions indicated by the diurnal *barometric* changes are those which are responsible, in the manner proposed by the above theory, for the daily magnetic changes. In his second memoir this hypothesis was carefully examined, using the data of his former investigation as the basis of discussion. The general conclusion was favourable to the theory, although attention was drawn to several features of the phenomenon which remained difficult to explain.

Among other works on the subject which have appeared since the publication of SCHUSTER'S earlier memoir, those by FRITSCHE,* G. W. WALKER,† and VAN BEMMELEN‡ may be noted here. The two former authors confined themselves to the solar diurnal magnetic variations, but VAN BEMMELEN broke fresh ground by applying harmonic analysis also to the lunar diurnal variations. The importance of the latter was fully recognized by both BALFOUR STEWART and SCHUSTER, who, in his second memoir (p. 181), urged the desirability of further study of them.

Some of the above, and other, writers reached conclusions adverse to the STEWART-SCHUSTER theory, partly owing to the fact that the results of their analyses of the observational data differed from SCHUSTER'S. The present paper embodies an attempt to resolve the points in dispute, and to remove other obscurities in the theory. New analyses are made both of the solar and lunar diurnal magnetic variations, so that the chief facts relating to each may be discussed together. It is taken as axiomatic, in view of the general resemblance between the two phenomena, that in the main the same theory and similar mechanisms must apply to each. Various modifications of SCHUSTER'S hypotheses and results are found to be necessary, but the essential points of the theory are confirmed by this investigation.

The discussion of the third and fourth, as well as the 24- and 12-hour, harmonics in the magnetic variations is one of the more novel features of this paper: previous discussions have generally been confined to the diurnal and semi-diurnal components, and doubts have been cast on the value of the higher frequency terms, which I hope the present investigation will remove. In the case of the lunar variations, only the semi-diurnal term has hitherto been used; this, however, was because, while other harmonics are present, their phases vary through a multiple of 2π throughout each lunar month, so that they disappear from the ordinary mean monthly variation calculated as it has been in the past. In an earlier memoir § I have shown how all four harmonics can be determined by computing the variations

* FRITSCHE, St. Petersburg, 1903, and Riga, 1905 and 1913 (these papers were apparently privately printed and circulated).

† G. W. WALKER, 'Roy. Soc. Proc.,' A, vol. 89, p. 379, 1913.

‡ VAN BEMMELEN, 'Meteorologische Zeitschrift,' 5, p. 218, 1912; 12, p. 589, 1913.

§ 'Phil. Trans.,' A, vol. 213, p. 279, 1913; and A, vol. 214, p. 295, 1914. Also see 'Phil. Trans., A, vol. 215, p. 161, 1915.

for groups of days all at the same lunar phase, afterwards correcting the phase of the resulting Fourier coefficients to the epoch of new moon. In this way a considerable similarity between the relative amplitudes and phases of the various components of the solar and lunar magnetic variations is revealed.

The changing phase of the non-semi-diurnal terms in the lunar variation is a result of the combination of a lunar semi-diurnal variation (a lunar atmospheric tide) with an effect depending on solar time. The latter is here identified with the variation in the electrical conductivity of the upper atmosphere, owing to some solar ionizing influence. At new moon the two effects are in phase, and the lunar magnetic variations resemble the solar; in the latter case, of course, both the atmospheric oscillation and the variable conductivity keep time with the one body, the sun.

SCHUSTER found that while the main cause of the solar diurnal variations was external to the earth, there was also an induced system of earth currents, partly neutralizing the vertical force variations. This result is confirmed here, though with numerical modifications. The external contribution to the horizontal force variations is estimated at about 2·5 times the internal, as against about four in his memoir; also whereas no phase difference between the two current systems was found, a difference of from 10° to 25° is here indicated. It is shown that the internal field could be produced through induction by the outer currents, provided that, beneath an upper non-conducting layer of 150 or 200 miles depth, the substance of the earth has a uniform specific resistivity of amount $2{\cdot}74 \,.\, 10^{12}$ C.G.S. A conclusion of this kind was arrived at in 1889 by SCHUSTER, and this investigation only modifies his in detail; he made no estimate of the resistivity of the inner core, and suggested 1000 km. as the depth of the outer layer.

The lunar diurnal magnetic variations are undoubtedly due solely to a semi-diurnal atmospheric oscillation. The relative magnitudes of the various harmonic components in the magnetic variation afford information regarding the conductivity of the atmospheric layers in which they are produced. It appears that the currents flow mainly in the sunlit hemisphere, the conductivity in the dark hemisphere being very small. Its diurnal variation can be approximately ascertained, and its maximum value numerically estimated; this proves to be higher than the value originally suggested by SCHUSTER. The phenomena of electric wave transmission also suggest the existence of such a layer of variable electric conductivity, and, in addition, of a still higher layer of constant and uniform conductivity. The magnetic variations give no indication of the latter layer.

The main terms in the solar magnetic variation are so similar, in most respects, to those of the lunar variation that they too would appear to be produced mainly by a semi-diurnal atmospheric oscillation. The 24-hour components are relatively larger, however, than in the lunar magnetic variation, so that, although the phases of the 24- and 12-hour terms show remarkably close agreement, there is ground for supposing that a diurnal atmospheric oscillation may be involved in addition.

The seasonal spherical harmonics in the magnetic variations are also considered, and it is found that they are relatively twice or thrice as large in the lunar variation as in the solar variation, in comparison with the harmonics which persist uniformly throughout the year. This suggests that the main 12-hour oscillation in the solar case may produce seasonal harmonics which are partly neutralized by some other oscillation, such as that of 24-hour period, just mentioned.

The phase relations of the latter oscillation present some difficulty, and it is questionable whether it is connected with the 24-hour barometric variation, of thermal origin, which is observed at the earth's surface. The 12-hour barometric variation, on the other hand, is so much more fundamental in character that it is not unreasonable to suppose that it persists even up to the high levels here contemplated. The magnetic data suggest that its proportional amplitude ($\delta p/p$) diminishes upwards to the extent of one-half its surface value.

The upper air currents responsible for the magnetic variations will have a heating effect which can be approximately calculated, and it appears that there is a possibility of the production, by this means, of an appreciable solar diurnal temperature and pressure variation in the conducting layer. It may be that the above-mentioned diurnal oscillation, peculiar to the solar diurnal magnetic variations, is to be accounted for in this way.

The questions of phase raised by this discussion prove to be very perplexing. As the theory would indicate, the phases of the annual mean harmonics of various periods agree amongst themselves, both in the solar and lunar variations. But these phases seem to possess little or no relation with those of the solar and lunar semi-diurnal *barometric* variations at the earth's surface. It would appear that the phase of the solar barometric variation diminishes with increasing height, by an amount which has been estimated at 80 or 90 degrees.* But a much larger change of phase with height is required, affecting the lunar as well as the solar barometric variation, if these are to be brought into simple relation with the magnetic variations.

The other principal remaining difficulty is that the diurnal North force variations do not agree at all well with those calculated from the potential function which represents the diurnal West force variations. This does not seem to be explicable, as G. W. WALKER has suggested, by the presence of magnetic variations depending on the time of some fixed meridian.

In order that the present paper may be the better understood, it has seemed well to indicate the existing state of the problem, by giving a brief critical account (§§ 2 to 6) of the work of the previous writers already named. In the course of this historical survey mention is made of the chief modifications of previous methods, and of previous conclusions, which are introduced in the present discussion.

The burden of labour entailed by an investigation of this kind is very great, and could

* This was not remarked on in SCHUSTER'S second memoir, in which a measure of agreement seemed to be indicated between the phases of the barometric and magnetic variations.

not have been undertaken without considerable assistance. The heaviest part of the arithmetical work consisted in the computation of the lunar diurnal variations (§ 12); no reduced data of the desired kind were available, so that the variations had to be newly computed from the published hourly values of the magnetic elements, seven years' records from each of five observatories being used. Skilled assistance was obtained in this and all the other work of computation, wherever possible. In regard to this I wish to make grateful acknowledgment of the help placed at my disposal by the Government Grant Committee of the Royal Society, by Dr. SCHUSTER, and by the Astronomer Royal. Also in the preparation of the data of Tables I. to III. relating to the solar diurnal variation, I am indebted to the Astronomer Royal for computing assistance, and to Mr. W. W. BRYANT, Superintendent of the Magnetic and Meteorological Department at the Royal Observatory, Greenwich, who personally shared in and controlled the reduction of the published data to this form.

I wish also to acknowledge the courtesy of the following directors of observatories, who furnished me with manuscript records of such of their observations as I had need of, and which were at the time unpublished: Dr. ANGENHEISTER (Samoa), Mr. P. BARACCHI (Melbourne), Dr. SCHULZE (Pilar and Laurie Island), and Mr. SKEY (Christchurch, N.Z.).

The preparation of even the tables of initial data, such as Tables I. and II. and (much more) IV. to VI., is a task of considerable magnitude, which I believe must put a serious obstacle in the way of further advance in the subject. In order that future workers may not be repelled by such initial difficulties, it is very desirable that directors of magnetic observatories should reduce their own observations more completely, and publish them in a readily available form. With regard to the lunar diurnal variations I have already made suggestions as to the manner in which this might advantageously be done.* The present discussion of these variations seems to confirm the desirability of further investigation. The same applies, though to a smaller extent, to the solar diurnal variations. In the latter case I would recommend that a threefold sub-division of the year should be adopted, as for the lunar diurnal variations in § 12; and that the variations from quiet days only for the mean of a number of years, should be used.

PART I.—THE PRESENT STATE OF THE PROBLEM.

§ 2. SCHUSTER'S *first investigation* (1889).

The data used by SCHUSTER were the Fourier coefficients of the first four harmonics in the solar diurnal variations of North, West, and vertical magnetic force, taken separately over the summer and winter halves of the year 1870, from the observations at Bombay, Lisbon, Greenwich, and St. Petersburg. The year 1870 happened to be

* 'Phil. Trans.,' A, vol. 214, p. 295, 1914.

a year of abnormally high solar and magnetic activity,* so that the data were not typical, though this is no drawback so far as the investigation is self-contained. The first part of the discussion dealt with the determination of a potential function which, on differentiation, yields as close a representation as possible of the observed variations in the North and West components of force. It was provisionally assumed that such a potential function exists, and that it is symmetrical with respect to the earth's axis and remains constant in its relation to the sun as the earth revolves—*i.e.*, that the variations depend solely on local time. Moreover, since the number of stations from which data were used was small, it was necessary to assume symmetry also with respect to the equatorial plane. Thus the Northern winter variations were taken to represent† the variations, at corresponding Southern latitudes, contemporaneous with the observed Northern summer variations.‡ The period of the year to which the calculations apply is consequently a half-year centred at either solstice.

If such a potential function exists, data from the above four stations should suffice to indicate its main features, although the area of the earth's surface from which they are drawn is somewhat limited. It is desirable to have more stations, however, both in order to test the validity of the assumption, and to evaluate the function more exactly. WALKER states that the observed variations indicate the presence of important harmonics not depending solely on local time; the new data of this paper do not give much support to this conclusion, but they agree with FRITSCHE'S results in suggesting that an appreciable part of the variations at any one station is local and peculiar to the place. If data from only a few observatories are used it is nearly always possible to represent them closely by including a sufficient number of tesseral harmonics in the potential function; but from the above it is clear that only the main terms are likely to have significance as indicative of variations which are world-wide. For this reason it would now seem that SCHUSTER'S analysis of his data was unnecessarily elaborate, but at the time there was no previous experience to serve as a guide, and it was probably best to risk going too far in this direction, rather than to fall short of what the data might yield. Only the more important terms, however, which agree with those considered in this paper, were discussed. They are reproduced (in the notation of § 9) in Table D, p. 25, for comparison with other determinations by FRITSCHE and the present author. The various investigations give results which agree fairly well as regards the phase angles, and in the relative orders of magnitude of the various amplitude-coefficients. The absolute

* WOLFER'S sunspot number for this year was 139, a value which has been approached only on one other known occasion, viz., in 1848, when the sunspot number was 124.

† With suitable changes of sign in certain components.

‡ This was done only for the first two components of periods 24 and 12 hours. The seasonal terms were not considered in the case of the 8-hour and 6-hour components, for which the mean of summer and winter was taken to apply to each hemisphere alike.

magnitude of SCHUSTER's results is in nearly every case greater than those here obtained, even for the year of sunspot maximum. This was to be expected in view of the exceptional character of the year 1870.

Having obtained a potential function which would account for the horizontal force variations, SCHUSTER compared the observed vertical force variations with those calculated from this function on the respective hypotheses that its origin was (*a*) external, (*b*) internal to the earth. Only from one of the four stations (Lisbon), unfortunately, were satisfactory vertical force data available for the year 1870. For the other three observatories data relating to later years had to be taken.*

It appeared that the *phase* of the observed vertical force variations agreed completely with the assumption of an external cause (and was therefore opposite to that corresponding to the second hypothesis), but that the observed amplitude was only about half the calculated amplitude. This was explained by supposing the primary varying magnetic field, above the earth's surface, to be accompanied by a secondary field within the earth due to electric currents induced by the primary field. The secondary field would reinforce the horizontal force variations due to the primary, and would partly neutralize the vertical force variations. But an accompanying phase difference was to be expected between the calculated and observed vertical force results, and this appeared not to exist. Certain researches by LAMB,† indeed, indicated that if the earth were assumed uniformly conducting, a reduction of the amplitude of the vertical force variations by one-half (as in the Lisbon data) should be accompanied by a phase change of about 40 degrees. This difficulty was surmounted, however, in pursuance of a suggestion by LAMB, by assuming that the conductivity of the inner core of the earth exceeds that of the upper layers. In his second memoir (p. 169) SCHUSTER roughly estimated the thickness of the outer non-conducting crust to be about 1000 km.

The general conclusion as regards the diurnal and semi-diurnal components of the solar diurnal magnetic variation was that the potential of the external field agrees in phase with, but is four times the magnitude of, that for the internal field.‡ The components of shorter period were hardly at all discussed in either of his memoirs. The character of the vertical force data used, and the presence of local irregularities in the variations at single stations, naturally suggest that this separation of the internal and external fields may be somewhat uncertain. FRITSCHE's and VAN BEMMELEN's results are considerably different, and those also of the present paper, while confirming SCHUSTER's main conclusions, differ from his results in some important respects.

* As regards Greenwich and St. Petersburg this was because the temperature corrections to the vertical force records were not properly known in 1870. The vertical force magnetograph at Bombay did not come into operation till after 1870.

† LAMB, 'Phil. Trans.,' 1883, p. 536; also the appendix to SCHUSTER's memoir of 1889.

‡ *Cf.* p. 170 of the second memoir: this conclusion was not explicitly stated in the first memoir.

§ 3. FRITSCHE'S *Investigations of the Solar Diurnal Magnetic Variations.*

FRITSCHE'S investigations of the solar diurnal magnetic variations are works of considerable magnitude and numerical detail, and are contained in three papers of 1902, 1905, and 1913.* In the first paper a Gaussian potential function is determined to represent as closely as possible the variations in the North and West components of force at 27 stations. The function was determined separately for the summer and winter half-years. It was not necessary to assume symmetry about the equator, as SCHUSTER had done, since FRITSCHE'S stations included five in the Southern hemisphere; the assumption that the variations depend only on local time was, however, retained. The use of a much larger number of stations was in itself an improvement, but this was attained, in FRITSCHE'S case, by throwing over an important consideration to which SCHUSTER rightly gave much weight, viz., that the data should all relate to the same epoch. FRITSCHE'S data are drawn from series of observations extending in some cases only over a few months, and in others over several years, and their epochs range from 1841 to 1896. They are consequently far from homogeneous, both on account of the eleven-year cycle in the magnetic activity of the earth, and probably also because of varying degrees of observational accuracy.

Of the 27 stations, nine were North of latitude 60° N., thirteen lay between 0° and 60° N., while the remaining five extended from 0° to 60° S. After deducing his potential functions from the North and West force data taken together, using the method of least squares,† FRITSCHE made an elaborate numerical comparison of the calculated and observed variations. The best agreement was found for the ten Northern observatories lying between the tropical circle and 60° N.; it was less good for the tropical and Southern stations, and very bad for the nine stations above 60° N. latitude. The last circumstance is perhaps not unnatural, owing to the divergence of the magnetic from the geographical poles of the earth. So long as the analysis of the magnetic variation is based on the assumption that they depend solely on local time, it seems best to use data only from stations between ±60° latitude. All the other investigations described here conform to this rule, and FRITSCHE himself decided later that it was desirable to exclude the nine polar stations and to re-calculate the potential function from the remaining 18 observatories. This work is described in his 1913 paper. The 18 sets of data, combined

* FRITSCHE, St. Petersburg, 1902, Riga, 1905, and Riga, 1913. The second paper, so far as it deals with the daily variation, is in the nature of an appendix to the first, and need not be separately considered.

† The 27 stations were combined into six groups, and the mean diurnal inequalities in each element were computed from those for the separate stations in each group. These mean inequalities (in the form of 24 hourly values) were harmonically analysed, and the Fourier coefficients were then treated by the method of least squares so as to fit a potential function to them as closely as possible. Two of the six groups included the stations North of 60° N.

into four groups only (as they were given in the 1902 paper), were treated by the method of least squares as before. While the resulting potential functions were not greatly different from those first obtained, the residuals for the given 18 stations were improved, without much affecting those for the other nine. The chief terms in the re-calculated functions are exhibited in Table D, § 10, in a form allowing comparison to be made with SCHUSTER'S and the new results of this paper. FRITSCHE'S values, like those here found for the years 1902 and 1905, are much less than SCHUSTER'S, for the reason already mentioned. But the agreement is more striking than the differences, considering the different material, epochs, and methods of analysis used in the various investigations.

FRITSCHE treated the vertical force data from the same stations in a precisely similar way, and thus deduced from them a potential function which was independent of the horizontal force variations. In Table E, p. 26, the re-calculated results of his 1913 paper are compared with the corresponding results obtained in this paper. The two sets of figures generally agree in sign, but numerically the agreement is much less good than for the horizontal force data of Table D. I can only attribute the difference to the greater difficulty of obtaining reliable observations of the *vertical* force variations.* This renders it very necessary to use only the most modern and reliable data available.

The calculation of a separate potential function from the vertical force data makes possible a more satisfactory estimation of the respectively external and internal parts of the magnetic variation field than SCHUSTER'S limited material allowed. The method used by FRITSCHE, and also in this paper, is explained in §§ 8, 9, and only the results will be mentioned here. In place of SCHUSTER'S value, 4 : 1, the ratio of the surface potentials of the outer and inner portions of the field was given as 1·75 : 1 in the 1913 paper (in the 1902 paper the discordance was still greater, the stated ratio being 1·49 : 1). Another method used by FRITSCHE for estimating the same ratio gave the results 1·59 : 1 (1913) or 1·44 : 1 (1902). He concluded that the internal field was too nearly equal to the external field to allow it to be regarded merely as an induction product of the latter. It need hardly be stated how much the difficulties in the way of an explanation of the phenomenon would be increased if such a conclusion were substantiated; two independent mechanisms would then have to be co-ordinated and accounted for.

FRITSCHE did not discuss the phase relations of the internal and external potentials, nor did he consider SCHUSTER'S theory of a non-uniformly conducting earth. In view of the new analysis of improved data in this paper, it does not seem necessary to complete FRITSCHE'S discussion in this respect.

* An error of another kind which has to some extent affected earlier investigations of the present nature may be mentioned, viz., that by a mistake in the formulæ of reduction the Batavian vertical force variations have been recorded at twice their true amount from their commencement in 1880 until the discovery of the error in 1913 ('Batavian Observations,' 35, 1912, Preface).

It may also be noticed that the comparison between the calculated and observed variations in his paper showed a much more satisfactory agreement for the West component than for the other two. This seems to be partly a consequence of greater freedom from local irregularities in this element, and is confirmed by the results of the present investigation.

§ 4. G. W. WALKER's *Investigation* (1913).

The investigation by WALKER was confined within narrower limits than FRITSCHE's, both in respect of the data used and consequently also in their analysis. The data consisted of the annual mean solar diurnal variations of the North, West, and vertical components of force at nine observatories, and the components having periods of 24 and 12 hours were alone considered. The nine stations ranged in latitude from 60° N. to 6° S. While in every case the observations were of recent date, they did not all refer to the same year. It appeared that a potential function of simple type (Q_2^1 and Q_3^2 for the 24- and 12-hour periods,* respectively) could be fitted fairly well to either the West or North force data separately, but that the same numerical coefficient would not apply to both. This was taken to indicate that the assumption of a potential function, at any rate of one depending solely on local time, was invalid. SCHUSTER and FRITSCHE, using the two components together, and including a considerable number of harmonics in their analyses, did not notice such a discrepancy (*cf.*, however, the last paragraph of § 3).

WALKER tried to overcome this difficulty by introducing harmonic functions not dependent solely on local time, and in this way he obtained a better representation of his data. But the crucial test of the existence of such additional harmonics must consist of the examination of data from stations of widely different longitudes, and, unfortunately, seven out of the nine stations used by WALKER lay between 3° W. and 31° E. The data of the present paper, which had been collected before the publication of Mr. WALKER's paper, were chosen with a view to a decision upon this question,† and are from fairly widely distributed stations. While the simple potential functions of type Q_2^1, Q_3^2 do not well represent the North force data, the evidence for the existence of any considerable harmonics not depending on local time does not appear to be strong. For the components of period 12 hours or less, I am inclined to attribute the North force discordance to local irregularities, while leaving the question open in the case of the 24-hour component.

The terms in WALKER's representation which depend on local time and are symmetrical about the equator are compared in § 10 with the corresponding annual terms obtained by other writers. The phases agree well, and the amplitudes found

* These are the main annual terms of these periods which were found also in the other investigations summarized in Table D.

† In consequence of a suggestion made by SCHUSTER in his second memoir, p. 172.

by WALKER agree in order of magnitude with those tabulated, although his value for C_3^2 is rather small. Since his data refer to the mean of a year, and are drawn almost entirely from the Northern hemispheres, the unsymmetrical terms in his analysis are not comparable with any results of this paper.

With regard to the vertical force data, WALKER showed that the 24-hour terms in the horizontal force potential would fit the vertical force observations if it was assumed that the internal and external fields agree in phase, and that their amplitude ratio, in the case of the second degree harmonics, is that found by SCHUSTER, viz., 4 : 1; the internal contribution to the harmonic of the first degree was taken to be nil. The 12-hour component was examined in more detail, and it was estimated that the internal contribution to the harmonics of degree three was about one-quarter the external, and that a phase difference between the two would improve the agreement with the vertical force data; as before, the minor harmonic (in this case Q_1^1) was assumed to be entirely external. The phase differences alluded to amounted to 35 degrees in the case of Q_3^2, and 54 degrees in the case of Q_3^3, the internal field being in *advance* of the external. FRITSCHE's data indicate a phase difference of smaller amount in the contrary sense. The theoretical significance of these differences was not considered, perhaps because the phase difference seemed to be absent in some cases and present in others. The data of the present paper indicate that in all the important, well-determined harmonics, both in the solar and lunar diurnal variations, the phase of the internal field is in advance of the external phase by amounts of the order of 20 degrees. In §§ 15–17 it is shown how these phase differences and the amplitude-ratios can be accounted for by a modified form of SCHUSTER's hypothesis of a non-uniformly conducting earth.

§ 5. VAN BEMMELEN's *Study of the Lunar and Solar Semi-diurnal Magnetic Variations* (1912).

While the lunar diurnal magnetic variation has often been studied, and from many points of view, VAN BEMMELEN was the first to investigate it as a world-wide phenomenon after the manner of SCHUSTER and FRITSCHE. His data consisted of the Fourier coefficients a_2, b_2* for the lunar semi-diurnal variations of the geographical components of magnetic force, taken separately over the summer and winter half-years, from fifteen observatories. The latitude range of these was 60° N. to 43° S. The material was somewhat heterogeneous, relating to different epochs, and calculated in different ways from unequal periods of observation; but a careful

* The first harmonic coefficients a_1, b_1 were also calculated and tabulated, but it was stated that they were irregular, and probably not a real part of the phenomenon. This is the case when they are calculated directly from the mean of a month, as for the semi-diurnal variation. A later paper ('Phil. Trans.,' A, vol. 213, p. 279, 1913) showed, however, that a real 24-hour component exists, which can be calculated only by separately considering the days of different lunar phase.

attempt was made to reduce the data to a common standard, so that the consequent drawback is less than in FRITSCHE's investigations. Besides using various published data, new reductions were made for several stations, and the paper is valuable on this account as well as for its main purpose. For the present paper it was unfortunately necessary to re-calculate the lunar variation for some of these stations so recently dealt with by VAN BEMMELEN, since the harmonics of varying phase could not otherwise be determined. The circumstance does, however, render possible an interesting comparison (§ 14) between the results of the two sets of computations.

Although his data referred separately to the summer and winter half-years, VAN BEMMELEN discussed only the mean annual values, neglecting the seasonal variations. He concluded that the horizontal force variations had a potential, which he determined (after trial of SCHUSTER's method) by the method of least squares; unlike FRITSCHE, however, he used the separate values of a_2 and b_2 from each station instead of combining them into groups, and apparently, also, only the West force data were used. The vertical force data were similarly treated, and a separation of the internal and external parts of the lunar magnetic variation field was then effected.

In a correcting paper of 1913 this calculation was revised, since the "least squares" method of determining the potential seemed to give too much weight to some rather irregular data of early epoch from the three Southernmost stations—the Cape of Good Hope, Melbourne, and Hobarton.* SCHUSTER's method was returned to as enabling more discrimination to be exercised between the various data in the course of the work. The resulting analysis was perhaps somewhat over-elaborate, but the principal harmonic, the one dealt with also in this paper, agrees moderately well with the result here obtained (*cf.* § 14). The original calculation had made it appear that the external variation field was actually *less* than the internal field; the revised paper reversed this conclusion, although the inner field was given as more nearly approaching the outer field, in magnitude, than the new analysis of this paper would suggest. VAN BEMMELEN, indeed, as the result of his calculations, still contemplated the possibility of a primary inner as well as a primary outer field.

In his first paper he had also attempted to bring the lunar semi-diurnal variation into relation with the lunar semi-diurnal barometric variation (as observed at Batavia), just as SCHUSTER had done for the solar diurnal variations in his 1907 memoir. The discrepancy between SCHUSTER's and FRITSCHE's analyses of the solar diurnal magnetic variation, which were both discussed by VAN BEMMELEN, rendered the conclusions somewhat indefinite, and they must, in any case, have been superseded after the revised calculation of the lunar diurnal variation potential. In his second paper VAN BEMMELEN avoided the ambiguity just alluded to, by making

* Certain mistakes of sign had also been made in the first investigation, which were corrected. In the original paper the Bombay data were given as of thrice their true value, apparently through a numerical slip in the reductions, but on account of their discrepancy with other results they were excluded from the discussion.

a new determination of the semi-diurnal part of the solar diurnal variation potential. The data used were the a_2, b_2 coefficients of the annual mean solar diurnal variations in the three geographical components of magnetic force from fifteen observatories (nine North and six South of the equator). The epoch for most of the stations was 1901, a year of minimum solar activity; in other cases the data were corrected so as to correspond to such a year. In § 10 the main symmetrical annual term in the horizontal force potential is compared with the corresponding results from other investigations. The agreement in phase is good; the amplitude determined by VAN BEMMELEN is a little smaller than in most of the other cases. VAN BEMMELEN'S analysis also includes a strong unsymmetrical element Q_2^2, which is somewhat surprising, considering that it relates to the *annual* mean variation. The present results do not seem to suggest much asymmetry between the two hemispheres.

In attempting to separate the internal and external parts of the field, Dr. VAN BEMMELEN remarked on the hazardous nature of the task, owing to the "strong irregularities" in the vertical force data. It appeared, as the result, that the internal potential for the solar semi-diurnal variation was equal to, or even slightly in excess of, the external potential. This differs so greatly from my own conclusion (and also from that of FRITSCHE) that I have carefully compared his and my vertical force data. The figures for the nine Northern stations in common were closely similar in the two cases, but the Southern data were far from accordant. Only Batavia was common to the two sets of Southern observatories; during 1901 and 1902 the reorganization of the magnetic work at Batavia and the transfer of the instruments to Buitenzorg interrupted the record, and the 1889 Batavian observations, corrected to 1901, were used. As the correction noted in § 3 had not then been discovered, however, the a_2, b_2 coefficients as used have twice their true value. As regards the other Southern observatories, the date of three sets used, St. Helena, Cape of Good Hope, and Hobarton, was 1843, and the two latter series are very discordant. It would seem that too much weight has been given to the six Southern sets of vertical force data, and that here lies the explanation of the above discrepancy between the two separations of the external and internal variation fields.

§ 6. SCHUSTER'S *Second Memoir* (1907).

The theory of the diurnal magnetic variations originally propounded by BALFOUR STEWART (§ 1) was shown by SCHUSTER, in his first memoir, to be so far correct in that the main part of these variations arises from electric currents circulating above the earth's surface. BALFOUR STEWART'S theory also involved the hypothesis that the electromotive forces which impel these currents are supplied by the permanent terrestrial magnetic field acting on masses of conducting air which, in their bodily motion, cut through the earth's lines of magnetic force. In this hypothesis two important factors were unspecified, viz., the atmospheric motions and the atmospheric conductivity. In his second memoir SCHUSTER made definite suggestions on these points, and examined their consequences in connection with the results of his first memoir.

It may be stated at the outset that the direct magnetic effect of the convective motion of masses of electrified air was examined and found to be negligible (*loc. cit.*, § 10). Also it was shown (*loc. cit.*, § 8) that the horizontal magnetic field of the earth, and the vertical atmospheric motions, might be neglected, so that the investigation was concerned with the determination of the electromagnetic effect of a *horizontal* oscillation of the atmosphere, acting on the known *vertical* component of the earth's field. Initially the electrical conductivity of the upper atmosphere, where the currents flow, was supposed uniform and constant; afterwards examination was made of the modifications introduced into the theory by assuming the conductivity to be variable.

It was first proved that the atmospheric oscillations necessary for the production of the diurnal and semi-diurnal* magnetic variations (the conductivity being uniform) are of types Q_1^1 or Q_3^1 and Q_2^2 or Q_4^2 respectively. These are the types of motion indicated by the diurnal and semi-diurnal barometric variations. The theory that the latter variations are closely connected with the daily magnetic variations had already been tentatively advanced in SCHUSTER's first paper; he now submitted it to a detailed numerical test. The main difficulty confronting the theory was that the ratio of the diurnal to the semi-diurnal term in the magnetic variation (C_2^1/C_3^2) is very much greater than the corresponding ratio (c_1^1/c_2^2) in the barometric variation. The former ratio was found in his first paper to be 9·6,† while the calculated ratio was only 2·6; the latter calculation assumed the atmospheric conductivity to be uniform. As regards phase, the calculated variations lagged behind the observed by about $1\frac{1}{2}$ hours (or from $2\frac{1}{2}$ to 3 hours, on taking self induction into account).

In the above, the effect of barometric terms of type Q_3^1 and Q_4^2 was neglected, only Q_1^1 and Q_2^2 being considered. The nature of the diurnal term in the barometric variation is not known very definitely, however, and it was pointed out that by representing it in part by an oscillation of type Q_3^1 the amplitude ratio 2·6 could be increased: also that such an oscillation may be present in the upper layers of the atmosphere, which do not greatly affect the barometer, even if it is not found in the surface variation.

The term Q_3^1 would be called on to a smaller extent if the atmospheric conductivity is not uniform, but varies with the zenith distance (ω) of the sun. In this case the 12-hour oscillation Q_2^2 would contribute to the 24-hour magnetic variation Q_2^1, and the 24-hour oscillation Q_1^1 to the 12-hour magnetic variation Q_3^2, but the effect would be much more marked in the former case than in the latter. It was shown, in fact,

* Variations of higher frequency were not considered.

† In equation (10) of the second memoir the coefficient of ψ_3^2 should be 9·23 in place of 11·16 (this is the coefficient of ψ_2^2 as found in the first memoir, p. 486).

that assuming the variation of conductivity to follow the law $1+\cos\omega$,* the ratio 2·6 would be increased to 4·7 without drawing at all on Q_3^1.

As regards the seasonal change in the magnetic variations, it was stated that the large increase in summer could not be explained completely by the above variation of conductivity, and a cumulative seasonal change was suggested as a possibility, in addition to the variation with ω. The weight of this difficulty, however, was chiefly thrown upon the uncertainties in the atmospheric motions. In this paper the problem is simplified by the evidence afforded by the lunar diurnal variations, which indicate how largely a semi-diurnal oscillation is able to account for the 24-hour magnetic variations, owing to a much more marked variation of conductivity, between day and night, than that represented by the formula $1+\cos\omega$.

SCHUSTER estimated the order of the electric conductivity required by the theory, and discussed how far the high value thus found was physically possible or probable. He concluded that it was a possible value, which might perhaps be accounted for by ascribing the conductivity to the ionizing action of ultra-violet radiation from the sun. But it was remarked that the absorption of such radiation in the solar atmosphere might render this suggestion invalid.

The theoretical calculations of the paper dealt mainly with that part of the permanent magnetic field of the earth which is symmetrical about the geographical axis. It was pointed out, however, that the obliquity of the magnetic axis should result in the production of magnetic variations not depending solely on local time, and a search for these terms was suggested as a promising line of further work.

PART II.—A NEW ANALYSIS OF THE SOLAR DIURNAL MAGNETIC VARIATION.

§ 7. *Description of the Data.*

The data used in this investigation consist of the Fourier cœfficients a_n, b_n in the harmonic formula

(1) $$\Sigma\,(a_n \cos nt + b_n \sin nt)$$

for the solar diurnal variations in the North, West and vertical components of magnetic force. Results from twenty-one observatories are utilised, of which fifteen are North and six South of the equator, between latitudes ± 61 degrees. The average number of stations represented in any particular section of the final results is slightly less than twenty, however, since data in every element were not available from quite all the selected observatories at the chosen epochs.

The stations were selected so as to obtain as wide a distribution in longitude as the available records allowed (*cf.* § 27). Particulars of their names and positions are given in Table A. For convenience in the subsequent numerical analysis they have been divided into nine groups, as indicated.

* According to this law the conductivity evidently varies from a maximum at the point directly beneath the sun to zero at the antipodal point.

TABLE A.—The Sources of the Data Used for the Investigation of the Solar Diurnal Magnetic Variation.

No. of group.	No. of observatory.	Observatory.	Latitude (North +).	Co-latitude θ.	Longitude from Greenwich (East +).
			° ′	° ′	° ′
I.	1	Pavlovsk	59 41	30 19	30 29
	2	Sitka	57 3	32 57	−135 20
	3	Ekaterinburg	56 50	33 10	60 38
		Mean	57 51	32 9	—
II.	4	Potsdam	52 23	37 37	13 4
	5	Irkutsk	52 16	37 44	104 19
	6	Greenwich	51 29	38 31	0 0
		Mean	52 3	37 57	—
III.	7	Pola	44 52	45 8	13 51
	8	Tiflis	41 43	48 17	44 48
		Mean	43 17	46 43	—
IV.	9	Baldwin	38 47	51 13	− 95 10
	10	Cheltenham	38 44	51 16	− 75 50
		Mean	38 46	51 14	—
V.	11	Zi-Ka-Wei	31 12	58 48	121 26
	12	Honolulu	21 19	68 41	−158 3
		Mean	26 16	63 44	—
VI.	13	Bombay	18 54	71 6	72 49
	14	Vieques, Porto Rico	18 9	71 51	− 65 26
	15	Manila	14 35	75 25	120 58
		Mean	17 13	72 47	—
VII.	16	Batavia	− 6 11	96 11	106 50
	17	Samoa	−13 48	103 48	−171 46
		Mean	−10 0	100 0	—
VIII.	18	Pilar	−31 41	121 41	− 63 51
	19	Melbourne	−37 50	127 50	144 59
	20	Christchurch	−43 32	133 32	172 37
		Mean	−37 41	127 41	—
IX.	21	Laurie Island, South Orkneys	−60 45	150 45	− 45 1

The epoch of the data used is modern, the years 1902 and 1905* being chosen; these were years of sunspot minimum and maximum in their eleven-year cycle, two such periods being considered in order that the influence of solar activity on the phenomenon might be definitely determined. Also, so that the seasonal changes might be studied, each year was divided into four quarters of three calendar months each, beginning with February, March and April as the spring quarter. In calculating the mean variation for each of these eight periods, all days were used except a very few which were so highly disturbed as of themselves to be able to modify the quarterly means appreciably. The published data in most cases gave the variations of horizontal force and declination instead of North and West force, to which they had to be transformed. In some cases changes of phase had also to be made, to reduce the data to the adopted time-origin, which is here the local mean time of noon at each station. In the formula (1), t represents local time reckoned in angle at the rate of 15 degrees per hour. The first *four* harmonics ($n = 1, 2, 3, 4$) have been used throughout, the coefficients a_n, b_n being expressed in force units of amount 0·1 γ (10^{-6} C.G.S.), and reckoned positive to North, West, and radially (or vertically upwards). The initial data of this paper, relating to the *solar* diurnal magnetic variations, are given at the end of the discussion in Tables I. and II. (α), (β), (1)–(4), (pp. 74, *et seq.*).

§ 8. *General Outline of the Analysis of the Data.*

The data in the Tables I. and II. exhibit a considerable degree of regularity and of constancy in type; thus, save for the increased amplitude in the later year, the 1902 and 1905 values are closely similar: they show a general independence of longitude: and they are nearly symmetrical, or anti-symmetrical, with respect to the equatorial plane. These features are little less apparent in the terms of lower than in those of higher frequency.

Besides this, however, there is an irregularity about the numbers which seems to represent something peculiar to each station, persisting from year to year, and also affecting different elements unequally, the North component, perhaps, being the one most affected. In order to assist in eliminating this local part of the phenomenon from the analysis, nine group means of the data of Table I. have been formed, the groups being indicated and numbered in Table A. It would probably have been advantageous had each group included a still larger number of separate stations.

For the discussion of seasonal influence the method adopted was as follows: the mean of the spring and autumn data was taken to represent the main part of the phenomenon at the equinoxes, at which times there is a general similarity between

* Or rather 1902, February, to 1903, January, and similarly for 1905. It may also be noted that the Batavian vertical force data for the 1902 summer quarter are drawn from June and July observations only, no records being available for May. WOLFER'S sunspot numbers for these years were 5 (1902) and 64 (1905).

the variations in the Northern and Southern hemispheres. The summer and winter data are, of course, widely different, though the Northern summer bears considerable resemblance to the Southern winter, and *vice versa*. The mean, $\frac{1}{2}$ (Summer + Winter), was taken to represent the part of the phenomenon common to both seasons—it is, indeed, as would be expected, nearly symmetrical about the equator—while the semi-difference, $\frac{1}{2}$ (Summer − Winter), represents the variable, seasonal part of the phenomenon, which is anti-symmetrical about the equator. As will appear, the solstitial mean agrees very closely, in many respects, with the equinoctial mean, showing that a large part of the variation continues almost uniformly throughout the year. Even the residuals are very similar in the two cases.

These three sets of group means were taken separately for the years 1902 and 1905, and are to be found in Tables III. (α), (β), (γ). For the sake of completeness the equinoctial semi-differences, $\frac{1}{2}$ (Spring − Autumn), were also examined (*cf.* Table III. (δ)), but only for the mean of 1902 and 1905, as this set is less important than the others. It need hardly be pointed out that this analysis of the data can be immediately adapted so as to give the result for any single season, $\frac{1}{2}$ (Summer + Winter) − $\frac{1}{2}$ (Summer − Winter), for instance, giving the winter analysis alone.

In the investigation of these tables of group means, the aim kept always in view has been that of reproducing the broad features by as simple a mathematical representation as possible, considering, first of all, potential functions depending solely on local time. The residuals are discussed later, in connection with the question as to whether there are important terms varying with the longitude (§ 27).

On examination it became clear that the nearly constant, "annual" part of the phenomenon, corresponding to the equinoctial and solstitial group means of Tables III. (α) and II. (β), can be represented with fair accuracy by a single harmonic function for each periodic term, as far as regards the West force variations. The seasonal portions (Tables III. (γ) and III. (δ)) can, for the same component of force, be represented in each case by two such functions. These harmonic functions, depending on local time only, agreed in type with the main terms in SCHUSTER'S and FRITSCHE'S analyses, so far as the three investigations are comparable.

If the diurnal magnetic variations have a potential, the North force variations must be deducible from the potential function which represents the West force data. In Table III. the calculated values are given both for the West and North force variations, using the functions chosen to represent the West force data alone. The agreement with the observed values may be considered satisfactory in the latter case, but it is not nearly so good for the North force variations. Perhaps the local irregularities in the data will account for the discrepancy, at any rate in the case of the three components of shorter period (12, 8, and 6 hours). The residuals for the 24-hour component are very systematic, however, and could not be accounted for by a mere change in the amplitude of the potential function derived from the West force variations (*cf.* § 4). Possibly the two sets of data could be more nearly represented by the same potential function if more harmonics of higher degree were included. But I doubt whether any improvement thus made would be very substantial or of real value, and I have therefore judged it best not to discard the above simple and successful representation based on the West force variations alone. It may be added that if an independent attempt were made to determine, from the North force variations alone, the values of the harmonic functions present in them, of the type which represent the West force data, the results would differ little from those actually calculated from the latter.

The notation of the harmonic functions used in this analysis is described in § 9. In that notation a function

$$(2) \qquad (A_m^n \cos nt + B_m^n \sin nt)\, Q_m^n(\cos\theta) \qquad (n = 1, 2, 3, 4)$$

was used in Tables III. (α) and III. (β) to represent the variations ($a_n \cos nt + b_n \sin nt$) at the individual stations of various co-latitudes θ. The value of m in each case was found to be $n+1$. In Tables III. (γ) and III. (δ) two such functions, corresponding to $m = n$ and $m = n+2$, were used in each instance. The constants A_m^n and B_m^n are set out in Table C § (9).

As previous investigations have indicated, both inside and outside causes contribute to the magnetic field at the earth's surface, so that the vertical force variations cannot be deduced theoretically from the horizontal force variations. On the contrary, the potential function (if such exists), which represents and is calculated from the vertical force data alone, affords the means of separation of the respectively external and internal parts of the whole variation field. It proved on examination that functions (2), of precisely the same type as were used for the horizontal-force data, serve likewise for the vertical-force variations, only the numerical coefficients (denoted in this case by $\mathbf{A}_m^n$ and $\mathbf{B}_m^n$) being different. These also are given in Table C, alongside the values of A_m^n and B_m^n. The corresponding calculated values of a_n and b_n are given in Table III. for comparison with the observed data.

For the purpose of the subsequent discussion it was clearly advisable that A_m^n and B_m^n, $\mathbf{A}_m^n$ and $\mathbf{B}_m^n$ should be determined on some definite plan which would at least give results which were comparable in the different cases. Where only one harmonic function was involved in the representation of a given set of Fourier coefficients, as in Tables III. (α) and III. (β), the course adopted was very simple. The weighted mean of the various values of the function Q_m^n, corresponding to the mean latitude of each of the nine groups of observatories, was taken *numerically*, *i.e.*, negative values being treated as positive; the similarly weighted sum of the group mean values of a_n (or b_n) was also taken, the signs being reversed where this had been done for the calculated (negative) values. A simple division then gave the required coefficient A_m^n, B_m^n, $\mathbf{A}_m^n$, or $\mathbf{B}_m^n$. As regards the weighting, the Northern group means were each given unit weight, and the Southern means each half a unit of weight. The fifteen Northern observatories were thus given a total weight six, and the three, four, or five Southern observatories (according to the number available in the different cases) received a total weight one or one and a-half.

Where two harmonic functions were involved in the representation of one set of data, as in Tables III. (γ) and III. (δ), the same general method of weighting and combining the data was used, except that two equations had to be formed, to give the two coefficients. Usually the middle four or five values of a_n or b_n were used in one equation, and the remainder in the other.

§ 9. *The Harmonic Representation of the Magnetic Variation Field.*

A potential function which varies with the local time, but is otherwise the same at all stations along any parallel of latitude, can always be expanded in a series of spherical harmonic functions of the form

$$(3)\quad \psi_m{}^n \equiv \left\{\left(\mathrm{E}^n{}_{m(a)}\frac{r^m}{\mathrm{R}^{m-1}}+\mathrm{I}^n{}_{m(a)}\frac{\mathrm{R}^{m+2}}{r^{m+1}}\right)\cos nt+\left(\mathrm{E}^n{}_{m(b)}\frac{r^m}{\mathrm{R}^{m-1}}+\mathrm{I}^n{}_{m(b)}\frac{\mathrm{R}^{m+2}}{r^{m+1}}\right)\sin nt\right\}\mathrm{Q}_m{}^n(\theta).$$

Here $\mathrm{E}^n{}_{m(a)}$, $\mathrm{E}^n{}_{m(b)}$, $\mathrm{I}^n{}_{m(a)}$, $\mathrm{I}^n{}_{m(b)}$ are numerical coefficients; t is the local time reckoned in angle at the rate of 15 degrees per hour ($t = \lambda+t'$, where λ is the longitude measured towards the East from some standard meridian, and t is the time corresponding to that meridian); r is the distance from the earth's centre to the point considered, at colatitude θ (in this paper measured from the North pole as origin), and R is the earth's mean radius. The function $\mathrm{Q}_m{}^n(\theta)$, or $\mathrm{Q}_m{}^n$ as it will generally be written, is the ordinary tesseral harmonic of degree m and order n; it can readily be calculated from the formula

$$(4)\quad \mathrm{Q}_m{}^n = \frac{(2m)!}{2^m\,.\,m!\,(m-n)!}\sin^n\theta\left\{\cos^{m-n}\theta-\frac{(m-n)_2}{2^2\,.\,1!\,(m-\frac{1}{2})_1}\cos^{m-n-2}\theta\right.$$
$$\left.+\frac{(m-n)_4}{2^4\,.\,2!\,(m-\frac{1}{2})_2}\cos^{m-n-4}\theta-\ldots\right\},$$

in which the factors of the form p_s, where s is a positive integer, denote

$$p\,(p-1)\,(p-2)\ldots(p-s+1).$$

The part of (3) which depends on r^m is continuous and satisfies LAPLACE's equation within the sphere $r = \mathrm{R}$. In the case of the magnetic variation potential, consequently, it arises from an electric current system outside the earth. The remaining portion of (3), which depends on r^{-m-1}, similarly results from a current system within the earth. The letters E and I are chosen to indicate the respectively external and internal origins of the corresponding parts of the potential.

A term $\psi_m{}^n$ in the magnetic variation potential would lead to the following terms in the North, West, and vertical force variations at the surface of the earth $(r = \mathrm{R})$:—

(5) $$\frac{d\psi_m^n}{\mathrm{R}\,d\theta} = \{(\mathrm{E}^n_{m(a)}+\mathrm{I}^n_{m(a)})\cos nt + (\mathrm{E}^n_{m(b)}+\mathrm{I}^n_{m(b)})\sin nt\}\frac{d\mathrm{Q}_m^n}{d\theta} \qquad \text{(North)},$$

(6) $$\frac{1}{\mathrm{R}\sin\theta}\frac{d\psi_m^n}{d\lambda} = \{-(\mathrm{E}^n_{m(a)}+\mathrm{I}^n_{m(a)})\sin nt + (\mathrm{E}^n_{m(b)}+\mathrm{I}^n_{m(b)})\cos nt\}\frac{n}{\sin\theta}\mathrm{Q}_m^n \qquad \text{(West)},$$

(7) $$-\frac{d\psi_m^n}{dr} = -\{(m\mathrm{E}^n_{m(a)}-\overline{m+1}\,\mathrm{I}^n_{m(a)})\cos nt + (m\mathrm{E}^n_{m(b)}-\overline{m+1}\,\mathrm{I}^n_{m(b)})\sin nt\}\,\mathrm{Q}_m^n$$

(Radial, outwards).

These may be written in the form

(8) $$(\mathrm{A}_m^n \cos nt + \mathrm{B}_m^n \sin nt)\,\mathrm{N}_m^n(\theta) \qquad \text{(North)},$$

(9) $$(\mathrm{B}_m^n \cos nt - \mathrm{A}_m^n \sin nt)\,\mathrm{W}_m^n(\theta) \qquad \text{(West)},$$

(10) $$-(\mathbf{A}_m^n \cos nt + \mathbf{B}_m^n \sin nt)\,\mathrm{Q}_m^n(\theta) \qquad \text{(Radial, outwards)},$$

the new notation being thus defined :—

(11) $$\mathrm{A}_m^n \equiv \mathrm{E}^n_{m(a)}+\mathrm{I}^n_{m(a)}, \qquad \mathrm{B}_m^n \equiv \mathrm{E}^n_{m(b)}+\mathrm{I}^n_{m(b)},$$

(12) $$\mathbf{A}_m^n \equiv m\mathrm{E}^n_{m(a)}-(m+1)\,\mathrm{I}^n_{m(a)}, \qquad \mathbf{B}_m^n \equiv m\mathrm{E}^n_{m(b)}-(m+1)\,\mathrm{I}^n_{m(b)}.$$

The new symbols $\mathrm{N}_m^n(\theta)$ and $\mathrm{W}_m^n(\theta)$ are defined as follows :—

(13) $$\mathrm{N}_m^n(\theta) = \frac{d\mathrm{Q}_m^n}{d\theta}, \qquad \mathrm{W}_m^n(\theta) = \frac{n}{\sin\theta}\mathrm{Q}_m^n.$$

Table B contains a list of the particular values of the three functions Q_m^n, N_m^n and W_m^n corresponding to the special values of m and n with which we are

TABLE B.

$\mathrm{Q}_m^n(\cos\theta)$.	$\mathrm{N}_m^n = \frac{d}{d\theta}\mathrm{Q}_m^n$.	$\mathrm{W}_m^n = \frac{n}{\sin\theta}\mathrm{Q}_m^n$.
$\mathrm{Q}_1^0 = \cos\theta$	$\mathrm{N}_1^0 = -\sin\theta$	$\mathrm{W}_1^0 = 0$
$\mathrm{Q}_1^1 = \sin\theta$	$\mathrm{N}_1^1 = \cos\theta$	$\mathrm{W}_1^1 = 1$
$\mathrm{Q}_2^1 = 3\sin\theta\cos\theta$	$\mathrm{N}_2^1 = 3(2\cos^2\theta - 1)$	$\mathrm{W}_2^1 = 3\cos\theta$
$\mathrm{Q}_3^1 = \frac{3}{2}\sin\theta(5\cos^2\theta - 1)$	$\mathrm{N}_3^1 = \frac{3}{2}\cos\theta(15\cos^2\theta - 11)$	$\mathrm{W}_3^1 = \frac{3}{2}(5\cos^2\theta - 1)$
$\mathrm{Q}_2^2 = 3\sin^2\theta$	$\mathrm{N}_2^2 = 6\sin\theta\cos\theta$	$\mathrm{W}_2^2 = 6\sin\theta$
$\mathrm{Q}_3^2 = 15\sin^2\theta\cos\theta$	$\mathrm{N}_3^2 = 15\sin\theta(3\cos^2\theta - 1)$	$\mathrm{W}_3^2 = 30\sin\theta\cos\theta$
$\mathrm{Q}_4^2 = \frac{15}{2}\sin^2\theta(7\cos^2\theta - 1)$	$\mathrm{N}_4^2 = 30\sin\theta\cos\theta(7\cos^2\theta - 4)$	$\mathrm{W}_4^2 = 15\sin\theta(7\cos^2\theta - 1)$
$\mathrm{Q}_3^3 = 15\sin^3\theta$	$\mathrm{N}_3^3 = 45\sin^2\theta\cos\theta$	$\mathrm{W}_3^3 = 45\sin^2\theta$
$\mathrm{Q}_4^3 = 105\sin^3\theta\cos\theta$	$\mathrm{N}_4^3 = 105\sin^2\theta(4\cos^2\theta - 1)$	$\mathrm{W}_4^3 = 315\sin^2\theta\cos\theta$
$\mathrm{Q}_5^3 = \frac{105}{2}\sin^3\theta(9\cos^2\theta - 1)$	$\mathrm{N}_5^3 = \frac{315}{2}\sin^2\theta\cos\theta(15\cos^2\theta - 7)$	$\mathrm{W}_5^3 = \frac{315}{2}\sin^2\theta(9\cos^2\theta - 1)$
$\mathrm{Q}_4^4 = 105\sin^4\theta$	$\mathrm{N}_4^4 = 420\sin^3\theta\cos\theta$	$\mathrm{W}_4^4 = 420\sin^3\theta$
$\mathrm{Q}_5^4 = 945\sin^4\theta\cos\theta$	$\mathrm{N}_5^4 = 945\sin^3\theta(5\cos^2\theta - 1)$	$\mathrm{W}_5^4 = 3780\sin^3\theta\cos\theta$
$\mathrm{Q}_6^4 = \frac{945}{2}\sin^4\theta(11\cos^2\theta - 1)$	$\mathrm{N}_6^4 = 945\sin^3\theta\cos\theta(33\cos^2\theta - 13)$	$\mathrm{W}_6^4 = 1890\sin^3\theta(11\cos^2\theta - 1)$

concerned in this paper. These functions were numerically evaluated for each of the twenty-one values of θ in Table A, and the group means I. to IX. were formed; by means of these the coefficients A_m^n and B_m^n, $\mathbf{A}_m^n$ and $\mathbf{B}_m^n$ were determined as described in § 8, the West and vertical force Fourier coefficients being compared with the formulæ (9) and (10). The North force values of a_n and b_n were then calculated from A_m^n and B_m^n by means of (8). The observed and calculated values of a_n and b_n for the various components and seasons are tabulated in Tables III. (α) to (δ), and the values of A_m^n, &c., are given below in Table C.

It is clear from the Tables III. that the above harmonic analysis, although of a very simple character, gives a fair representation of the main features of the daily magnetic variation, except for the 24-hour component of the North force variation; for the other periodic terms of the latter variation the agreement with the potential calculated from the West force is better—perhaps even satisfactory, when the irregular "run" of the North force is considered.

§ 10. *Comparison with Previous Harmonic Analyses of the Solar Diurnal Magnetic Variation.*

It is of interest to examine how far the various studies of the solar diurnal magnetic variation, which have been made by different methods and with different data, agree in their main results. The principal terms in SCHUSTER'S, FRITSCHE'S (1913), and the present analyses, so far as they are comparable with one another, are collected in Tables D and E. In the former table the potential functions derived from the North and West force variations (or, in the present paper, from the West force variations only) are given. Instead of A_m^n and B_m^n, however, the amplitude C_m^n and phase α_m^n are given, where

$$(14) \qquad A_m^n \cos nt + B_m^n \sin nt = -C_m^n \cos (nt + \alpha_m^n).$$

All the results have been modified where necessary, to conform to the notation of this paper. In some cases the authors cited carried their analysis further than in the present instance, in others, as the table indicates, they stopped short of it; but the principal terms were in all cases the same.

In Tables D and E, the figures for the present paper, under the heading "annual terms," are obtained from the mean of the solstitial and equinoctial results in Table B. They therefore represent the mean for a whole year, as in the case of SCHUSTER'S and FRITSCHE'S results. The seasonal terms in the present analysis, on the contrary, refer to a half-year only, *i.e.*, the two quarters centred at the solstices. They may be expected to be of somewhat larger amplitude than if they had been derived from the two half-years, as in the previous discussions.

TABLE C.—The Spherical Harmonic Representation of the Solar Diurnal Variations of West and Vertical Magnetic Force.

Values of A_m^n, B_m^n, $\mathbf{A}_m^n$, $\mathbf{B}_m^n$.

The Unit of Force is 10^{-6} C.G.S.

n.	m.	Sunspot maximum, 1905.				Sunspot minimum, 1902.			
		West.		Vertical.		West.		Vertical.	
		A_m^n.	B_m^n.	$\mathbf{A}_m^n$.	$\mathbf{B}_m^n$.	A_m^n.	B_m^n.	$\mathbf{A}_m^n$.	$\mathbf{B}_m^n$.
Mean Equinox, $\frac{1}{2}$(Spring+Autumn).									
1	2	−54	26	−41	14	−34	25	−34	4
2	3	−7·2	3·7	−8·5	0·1	−5·1	3·6	−5·4	0·3
3	4	−0·45	0·40	−0·87	0·29	−0·31	0·37	−0·71	0·30
4	5	−0·0123	0·0203	−0·040	0·022	−0·0082	0·0156	−0·037	0·027
Mean Solstice, $\frac{1}{2}$(Summer+Winter).									
1	2	−52	22	−37	12	−31	22	−31	7
2	3	−6·4	3·1	−7·3	−0·5	−4·6	2·9	−5·6	0·6
3	4	−0·34	0·29	−0·71	0·30	−0·24	0·24	−0·46	0·23
4	5	−0·0086	0·0091	−0·020	0·010	−0·0039	0·0068	−0·015	0·013
Solstitial Inequality, $\frac{1}{2}$(Summer−Winter).									
1	1	−42	21	−4	−1	−40	17	−10	0
	3	−10·6	−4·5	−14·0	−2·5	−8·1	−2·3	−7·7	−3·5
2	2	−3·9	7·8	−1·5	2·6	−2·7	5·3	−3·4	2·5
	4	−0·60	0·35	−1·41	−0·37	−0·78	0·37	−0·85	−0·40
3	3	0·17	0·70	−0·038	0·070	0·01	0·63	−0·056	0·046
	5	0·036	−0·011	−0·030	−0·072	0·016	−0·006	−0·024	−0·051
4	4	0·048	−0·007	0·048	0·035	0·028	−0·010	0·030	0·040
	6	0·0045	−0·0099	−0·013	−0·040	0·0020	−0·0101	−0·013	−0·053
Mean of 1902 and 1905.									
		Solstitial inequality.				Equinoctial inequality.			
1	1	−41	19	−7	0	13	−13	0·61	−0·42
	3	−9·4	−3·4	−10·8	−3·0	−0·60	1·64	−0·99	1·49
2	2	−3·3	6·6	−2·4	2·6	0·2	−5·5	1·5	−3·2
	4	−0·69	0·36	−1·13	−0·38	−0·047	−0·104	−0·19	0·43
3	3	0·09	0·66	−0·047	0·058	−0·29	−0·60	0·00	−0·58
	5	0·026	−0·008	−0·027	−0·062	−0·0024	0·0039	−0·017	0·010
4	4	0·038	−0·009	0·039	0·038	−0·037	−0·013	−0·014	−0·037
	6	0·0032	−0·0100	−0·013	−0·046	0·0038	−0·0032	−0·013	0·009

TABLE D.—Comparison of the Potential Functions determined by SCHUSTER and FRITSCHE from North and West Force Data, with those here determined from West Force Data alone.

Annual terms.					Seasonal terms.				
Amplitude. — Phase.	Present paper. 1902.	Present paper. 1905.	SCHUSTER, 1870.	FRITSCHE.	Amplitude. — Phase.	Present paper. 1902.	Present paper. 1905.	SCHUSTER, 1870.	FRITSCHE.
C_2^1	40	58	89	59	C_1^1	43	47	54	59
α_2^1	35°	24°	24°	30°	α_1^1	23°	27°	27°	28°
C_3^2	5·8	7·6	9·2	6·2	C_3^1	8·4	11·5	10·1	6·2
α_3^2	35°	27°	31°	25°	α_3^1	344°	337°	311°	347°
C_4^3	0·41	0·53	0·63	0·42	C_2^2	6·0	8·7	11·2	8·4
α_4^3	47°	40°	67°	35°	α_2^2	63°	63°	75°	62°
C_5^4	0·0127	0·0180	0·0202	—	C_4^2	0·86	0·69	0·66	0·73
α_5^4	62°	55°	78°	—	α_4^2	25°	30°	− 18°	6°
					C_3^3	0·63	0·72	—	0·76
					α_3^3	91°	104°	—	88°
					C_4^4	0·030	0·49	—	0·33
					α_4^4	200°	188°	—	173°

The horizontal force harmonics in Table D show a considerable degree of agreement in regard to phase, especially for the main terms Q_2^1, Q_3^2, Q_1^1, and Q_2^2. The amplitudes obtained by SCHUSTER generally exceed those of FRITSCHE and the present paper, including those for the maximum sunspot year 1905. The year 1870 was, however, one of abnormally great solar and magnetic activity. Detailed numerical agreement between the various sets of results is not to be looked for, owing to this and other reasons, and it is satisfactory that the different determinations yield values of amplitudes and phases which show such general agreement.

The papers by WALKER and VAN BEMMELEN referred to in §§ 4, 5 contained the following results, which may be compared with the above :—

WALKER.	Annual terms.	$C_2^1 = 55$	$\alpha_2^1 = 25°$
	,, ,,	$C_3^2 = 4·3$	$\alpha_3^2 = 23°$
VAN BEMMELEN.	,, ,,	$C_3^2 = 4·3$	$\alpha_3^2 = 26°$

In Table E the coefficients $\mathbf{A}_m^n$, $\mathbf{B}_m^n$ obtained by FRITSCHE are compared with those of the present paper. The agreement for these vertical force results is much less good than for the horizontal force results in Table D. The reason for this is discussed in § 3 (*cf.* also § 5). The other authors quoted have not analysed the vertical force potential in a way which allows of a comparison with FRITSCHE'S and the present results.

TABLE E.—Comparison of the Potential Functions determined by FRITSCHE and in this Paper, from Vertical Force Data.

Annual terms.				Seasonal terms.			
	Present paper.		FRITSCHE.		Present paper.		FRITSCHE.
	1902.	1905.			1902.	1905.	
$\mathbf{A}_2^1$	−32	−39	−25	$\mathbf{A}_1^1$	−10	−4	−25
$\mathbf{B}_2^1$	6	13	31	$\mathbf{B}_1^1$	0	−1	−16
$\mathbf{A}_3^2$	−5·5	−7·9	−3·4	$\mathbf{A}_3^1$	−7·7	−14·0	−12·4
$\mathbf{B}_3^2$	0·5	−0·2	3·3	$\mathbf{B}_3^1$	−3·5	−2·5	0·0
$\mathbf{A}_4^3$	−0·58	−0·79	−0·66	$\mathbf{A}_2^2$	−3·4	−1·5	−3·9
$\mathbf{B}_4^3$	0·26	0·30	0·30	$\mathbf{B}_2^2$	2·5	2·6	5·6
				$\mathbf{A}_4^2$	−0·8	−1·4	0·1
				$\mathbf{B}_4^2$	−0·4	−0·4	−1·4

§ 11. *The Separation of the External and Internal Solar Diurnal Variation Fields.*

The equations (11) and (12) indicate how we may determine the respectively internal and external parts of the magnetic variation fields by means of the horizontal and vertical force potential coefficients A_m^n, B_m^n, $\mathbf{A}_m^n$, $\mathbf{B}_m^n$. Thus we have

$$(15)\qquad E^n_{m\,(a)} = \frac{(m+1)\,A_m^n+\mathbf{A}_m^n}{2m+1}, \qquad E^n_{m\,(b)} = \frac{(m+1)\,B_m^n+\mathbf{B}_m^n}{2m+1},$$

$$(16)\qquad I^n_{m\,(a)} = \frac{mA_m^n-\mathbf{A}_m^n}{2m+1}, \qquad I^n_{m\,(b)} = \frac{mB_m^n-\mathbf{B}_m^n}{2m+1}.$$

At the earth's surface ($r = R$) the value of the term ψ_m^n/R in the magnetic variation potential is (*cf.* (3))

$$(17)\qquad \begin{aligned}\psi_m^n/R &= \{(E^n_{m\,(a)}+I^n_{m\,(a)})\cos nt+(E^n_{m\,(b)}+I^n_{m\,(b)})\sin nt\}\,Q_m^n\\ &= -\{\mathbf{E}_m^n\cos(nt+\mathbf{e}_m^n)+\mathbf{I}_m^n\cos(nt+\mathbf{i}_m^n)\}\,Q_m^n,\end{aligned}$$

where in the last two lines the external and internal parts have been transformed in terms of their amplitudes and phases; these are connected with $E^n_{m\,(a)}$, &c., by the equations

$$(18)\qquad E^n_{m\,(a)} = -\mathbf{E}_m^n\cos\mathbf{e}_m^n, \qquad E^n_{m\,(b)} = \mathbf{E}_m^n\sin\mathbf{e}_m^n,$$

$$(19)\qquad I^n_{m\,(a)} = -\mathbf{I}_m^n\cos\mathbf{i}_m^n, \qquad I^n_{m\,(b)} = \mathbf{I}_m^n\sin\mathbf{i}_m^n.$$

The values of $\mathbf{E}_m^n$, $\mathbf{e}_m^n$, $\mathbf{I}_m^n$, $\mathbf{i}_m^n$, deduced from Table C by means of equations (15), (16), (18), (19), are given in Table F. For the solstitial as well as for the equinoctial inequality only the mean results for 1902 and 1905 are given.

TABLE F.—Amplitudes and Phases of the Spherical Harmonic Coefficients of the External and Internal Solar Diurnal Magnetic Variation Fields.

The unit is 10^{-6} C.G.S.

n.	*m*.	Sunspot maximum, 1905. External. $\mathbf{E}_m^n$.	External. $\mathbf{e}_m^n$.	Internal. $\mathbf{I}_m^n$.	Internal. $\mathbf{i}_m^n$.	Sunspot minimum, 1902. External. $\mathbf{E}_m^n$.	External. $\mathbf{e}_m^n$.	Internal. $\mathbf{I}_m^n$.	Internal. $\mathbf{i}_m^n$.
		Mean Equinox, ½ (Spring + Autumn).							
1	2	44·6	24°	15·4	29°	31·5	30°	11·5	53°
2	3	5·7	22	2·5	40	4·3	30	2·1	47
3	4	0·43	35	0·18	56	0·35	44	0·14	65
4	5	0·0167	52	0·0075	75	0·0135	55	0·0046	85
		Mean Solstice, ½ (Summer + Winter).							
1	2	41·2	22	14·8	25	28·8	30	9·7	50
2	3	5·0	20	2·2	39	3·8	27	1·7	45
3	4	0·33	35	0·12	55	0·24	42	0·10	53
4	5	0·0088	42	0·0038	57	0·0060	54	0·0019	78
		Mean of 1902 and 1905.							
		Solstitial inequality, ½ (Summer – Winter).				Equinoctial inequality, ½ (Spring – Autumn).			
1	1	32	23	13	30	12·5	225	5·9	226
2	2	5·1	61	2·2	69	3·9	264	1·5	262
3	3	0·46	87	0·23	119	0·46	292	0·21	305
4	4	0·025	182	0·015	212	0·025	333	0·015	352
1	3	7·3	341	2·7	338	1·52	72	0·50	76
2	4	0·53	17	0·27	48	0·048	348	0·094	270
3	5	0·016	220	0·014	178	0·0042	46	0·0010	61
4	6	0·009	264	0·003	199	0·0014	225	0·0039	213

As regards the reliability of the results in Table F, this is, of course, greater for the annual (*i.e.*, mean equinoctial and mean solstitial) terms than for the seasonal terms (solstitial and equinoctial inequalities). In the latter case the harmonics Q_m^n, where $m = n$, are fairly well determined, but the higher harmonics $m = n+2$ are much less certainly evaluated. Among the components of different periods the semi-diurnal one is probably most free from accidental error, but the agreements in phase and amplitude for the other periods in the various parallel cases seem to indicate that the harmonics Q^n_{n+1} and Q_n^n for all four periodic terms have definite terrestrial significance. It should be remembered that Q_m^n contains a numerical factor which increases rapidly with m (*cf.* Table B), so that the small amplitudes $\mathbf{E}_m^n$, $\mathbf{I}_m^n$ for the higher harmonics represent magnetic variations much less small, in proportion to the diurnal and semi-diurnal terms, than their numerical values suggest at first sight.

This completes the actual analysis of the solar diurnal magnetic variation field, although the original data, and the residuals between these and the values of a_n and b_n calculated from the analytical representation, will be discussed later in connection with the possible existence of a portion varying with the longitude. Before discussing the relation between the external and internal variation fields already determined, a similar analysis of the lunar diurnal magnetic variation will be described in order that the results of the two analyses may be considered together.

PART III.—A NEW ANALYSIS OF THE LUNAR DIURNAL MAGNETIC VARIATION.

§ 12. *Description of the Data and of the Method of Analysis.*

The data used in this analysis consist of the a_n, b_n Fourier coefficients in the analysis of the lunar diurnal magnetic variation according to the formula (1). But the time t in (1) is now local mean lunar time, reckoned at the rate of 15 degrees per mean lunar hour, which is approximately $\frac{29}{28}$ times as long as a mean solar hour. The time of origin is the local time of upper culmination of the moon, at the epoch of new moon. The conventions as regards the signs of the three geographical components of force are the same as in § 7. The unit of force in which the Fourier coefficients are expressed is 10^{-7} C.G.S., or $0{\cdot}01\gamma$, only one-tenth as large as the unit used in Part II.

The lunar diurnal magnetic variation is of very small amount, and it can be computed with any approach to accuracy only by the use of a long series of observations, so as to eliminate accidental errors arising from fortuitous disturbances of much larger magnitude than the variation itself. In the present case seven years' observations at each observatory have been used, and the years chosen were "quiet" as regards solar and magnetic activity. Except in the case of Batavia, the same seven years (1897 to 1903) were used for each station. Owing to the reorganization of the Batavian observatory during this period the years 1899 to 1901 had in this instance to be replaced by the correspondingly quiet years 1888 to 1890 of a previous solar cycle. A longer period than seven years would, of course, have been advantageous, but the labour of computation was already great.

The method of computation adopted, and in particular the method of calculation of the non-semi-diurnal harmonic components, of changing phase, has been described in an earlier paper, and a reference to this must suffice here.* In that paper are given the data, so obtained, from observations made at Pavlovsk and Pola. These are two of the five observatories chosen for consideration in this research; the other three are Zi-Ka-Wei, Manila and Batavia. The results obtained for the three latter have not hitherto been published; they are to be found in Tables IV. and V. The first of these contains the Fourier coefficients corresponding to the different phases of the moon, reduced to the epoch of new moon; these data are subject to certain corrections to amplitude and phase (*cf.* § 6 of the paper cited) which for convenience have been applied only to the mean results. The latter, transformed in terms of the geographical components of force, are given in Table V. The results for all the five observatories are collected in Tables VI. (a) to (d).

The method of treating the seasonal changes is slightly different from that adopted for the solar diurnal variations. Instead of dividing the year into four quarters it was divided into three equal parts, November to February representing the winter solstice, May to August the summer solstice, and the intervening four months the equinoxes. It would have been better, for purpose of comparison, if this method of sub-division had been adopted also for the solar diurnal variations; and, as Dr. CHREE points out, this sub-division of the year corresponds more closely than the one adopted with the actual seasonal changes in the solar diurnal variation.

As regards the solstitial data, since the semi-sum and semi-difference form the basis of analysis (*cf.* Tables VI. (c) and (d), corresponding to the solar diurnal Tables II. (β) and (γ)), the mean solstitial and the seasonal harmonics of Table G result from eight months' material, while the equinoctial material is based on only four months of the year. Less weight must accordingly be attached to the latter than to the former. It may also be remarked that the mean solstitial and equinoctial epochs are slightly different for the two sub-divisions of the year, and that therefore some allowance must be made for this when comparing the solar and diurnal results.

The analysis of the "observed" data of Tables IV. (a) to (d) is similar to that explained in § 8; owing to the small number of observatories dealt with, however, no

* 'Phil. Trans.,' A, 214, p. 295, 1914; *cf.* also A, 213, p. 279, 1913. It should be noted that in § 6 of the former paper a phase correction is given with the wrong sign, viz., $-2L/29$ degrees instead of (as it should be) $+2L/29$ degrees. In applying the correction it is to be understood that the time of lunar transit at Greenwich has been used as the local time of lunar transit on the *same civil day* at the other stations, otherwise 360 degrees would have to be added to or subtracted from L degrees. The phase angles given in Table VI. (a), p. 316 of the former paper, need to be diminished by 4·2 degrees (Pavlovsk) and 2·0 degrees (Pola) on the above account.

grouping was possible, so that the five observatories were each treated as were the groups in the former analysis. Equal weight was accorded to each of the five observatories.

Another divergence from the course described in Part II. was that the equinoctial inequality was not considered (with the adopted sub-division of the data, this was not possible). Also the harmonics Q^n_{n+2} in the solstitial inequality were left out of consideration, since the functions Q_n^n, as with the solar diurnal variations, represented the greater part of the seasonal change, and the data hardly sufficed to determine the small coefficients of the remaining harmonics Q^n_{n+2}.

§ 13. *Results of the Analysis of the Lunar Diurnal Magnetic Variation.*

The results of the harmonic analysis of the data in Tables VI. are collected in Table G, which includes also the values of the separated internal and external parts

TABLE G.—The Spherical Harmonic Representation of the Lunar Diurnal Variations of West and Vertical Magnetic Force, and of the Separated External and Internal Fields.

The Unit is 10^{-7} C.G.S.

n.	*m*.	West.		Vertical.		External E_m^n.		Internal I_m^n.	
		A_m^n.	B_m^n.	A_m^n.	B_m^n.	(*a*.)	(*b*.)	(*a*.)	(*b*.)
		Mean Solstice, $\frac{1}{2}$(Summer + Winter).							
1	2	8·8	28·5	− 5·0	23·3	4·3	21·8	4·5	6·7
2	3	0·4	7·4	− 6·0	4·6	− 0·6	4·9	1·0	2·5
3	4	0·16	0·42	0·00	0·79	0·09	0·32	0·07	0·10
4	5	0·0106	0·0106	0·005	0·048	0·0063	0·0102	0·0044	0·0005
		Equinox, Spring and Autumn.							
1	2	0·4	30·6	− 17·5	9·4	− 3·3	20·2	3·7	10·4
2	3	− 0·7	9·0	− 8·2	1·5	− 1·6	5·3	0·9	3·6
3	4	− 0·03	0·59	− 0·31	0·89	− 0·05	0·43	0·02	0·16
4	5	0·009	0·032	− 0·009	0·051	0·004	0·022	0·005	0·010
		Solstitial Inequality, $\frac{1}{2}$(Summer − Winter).							
1	1	− 9	57	− 21·4	5·5	− 13·1	39·8	4·1	17·2
2	2	− 3·3	22·2	− 6·1	5·5	− 3·2	14·4	− 0·1	7·8
3	3	0·23	1·68	− 0·42	1·02	0·07	1·11	0·16	0·57
4	4	0·009	0·022	0·010	0·039	0·006	0·016	0·003	0·0054

of the field. The notation is the same as that explained in §§ 9 and 11, only the seasonal divisions and the unit of force (here $0{\cdot}01\gamma$ in place of $0{\cdot}1\gamma$) being different. In Tables VI. (*b*) to (*d*) the calculated values of a_n and b_n, corresponding to these harmonic functions, are placed for comparison beside the values computed from the observational data.

The agreement between observation and calculation is naturally less good than for the solar diurnal variation, both because the accidental error in the data is greater in the present case (the whole effect being smaller) and because single observatories are here used in place of groups of observatories, so that the local irregularities are larger. The agreement is better, on the whole, in Tables VI. (*c*), (*d*) than in Table VI. (*b*), which rests on only half as much observational material as the two former; it is also better for the West than for the North component of force, as in Part II. The agreement is surprisingly good in Table VI. (*d*) for the horizontal force components. The vertical force variations are smaller than the horizontal force variations, and some of the values of $\mathbf{A}_m^n$ and $\mathbf{B}_m^n$, determined from the former, are very uncertain. On the whole, however, while the present data might be very considerably improved upon, the results prove more satisfactory than I had expected, at any rate for the horizontal force potential A_m^n and B_m^n. In judging the success of the analysis, regard may be had to the agreement of phase between the harmonic components of different periods and the reproduction of other features of the analysis of the solar diurnal magnetic variation, which show the close parallelism of the two phenomena. Although some of the tables of observed and calculated data in Table VI. do not seem to show much correspondence between the two, the results in Table G suggest that the assumptions underlying the analysis (*i.e.*, that the variations can be represented by the functions Q^n_{n+1} or Q_n^n) are sound, and that the discordances from the results of calculation arise from a relatively large amount of accidental error in the observational data.

The use of so small a number of observations is, of course, a fit ground for criticism, and calls for a repetition of this part of the investigation on a larger scale. The present is to be considered as merely a pioneer attempt. For this reason the analysis has been narrowly restricted, and the results must be discussed with due recognition that the percentage error is not small.

§ 14. *Comparison with* VAN BEMMELEN'S *Data.*

As bearing upon the question of the accidental error of the initial data of Tables VI. (*a*) to (*d*), it is interesting to compare the present values of a_2, b_2, with those calculated by VAN BEMMELEN for the same observatories (only the a_2, b_2 coefficients are given by the latter author). It should be borne in mind, however, that neither the epoch, the amount of observational material dealt with, nor the method of computation, was the same in the two cases. Dr. VAN BEMMELEN divided the year, for the purposes of his paper, into summer and winter halves, and in comparing with his results the summer and winter data of this paper, allowance has been made for this by incorporating with them the equinoctial results, with half weight. Table H contains all the material for comparison, and will sufficiently indicate the probable accuracy of our present knowledge of the lunar diurnal magnetic variation.

TABLE H (a).—Comparison of the Lunar Semi-diurnal Magnetic Variations at Five Observatories, as determined by VAN BEMMELEN (1) and in this Paper (2).

The Unit is 10^{-7} C.G.S.

Observatory.	West.				North.				Radial.			
	a_2.		b_2.		a_2.		b_2.		a_2.		b_2.	
	1.	2.	1.	2.	1.	2.	1.	2.	1.	2.	1.	2.
Summer.												
Pavlovsk . . .	110	112	− 18	5	56	5	58	86	− 19	− 2	− 9	− 3
Pola	137	122	− 9	33	85	51	117	103	16	34	− 55	− 34
Zi-Ka-Wei . .	191	210	− 68	23	30	9	9	6	111	112	− 18	27
Manila	119	132	− 47	− 21	− 10	− 22	− 29	− 60	− 8	20	− 78	− 86
Batavia. . . .	14	− 5	− 3	− 13	− 27	− 40	− 43	− 70	− 3	− 4	− 38	− 34
Winter.												
Pavlovsk . . .	− 5	8	14	18	− 35	− 12	− 28	11	− 1	3	− 15	1
Pola.	10	− 2	27	33	19	28	0	11	23	23	− 19	− 19
Zi-Ka-Wei . .	35	64	− 56	− 40	73	65	− 33	− 50	89	88	0	38
Manila	− 34	− 6	− 72	− 80	− 48	− 27	− 102	− 112	− 54	− 29	− 62	− 87
Batavia. . . .	− 178	− 195	− 4	− 11	− 49	− 66	− 60	− 99	− 3	− 10	13	10
Year.												
Pavlovsk . . .	53	60	− 2	12	11	− 3	15	48	− 10	0	− 12	− 1
Pola.	74	60	9	33	52	40	59	57	20	28	− 37	− 26
Zi-Ka-Wei .	113	137	− 62	− 8	52	37	− 12	− 22	100	100	− 9	32
Manila . . .	43	63	− 60	− 50	− 29	− 25	− 66	− 86	− 31	− 5	− 70	− 86
Batavia. . . .	− 82	− 100	− 4	− 12	− 38	− 53	− 52	− 85	− 3	− 7	− 13	− 12

TABLE H (*b*).—Comparison of the Lunar Semi-diurnal Magnetic Variations at Five Observatories, as determined by VAN BEMMELEN (1) and in this Paper (2).

The Unit is 10^{-7} C.G.S.

Observatory.	West.				North.				Radial.			
	c_2.		θ_2.		c_2.		θ_2.		c_2.		θ_2.	
	1.	2.	1.	2.	1.	2.	1.	2.	1.	2.	1.	2.
					Summer.							
Pavlovsk . . .	111	112	99°	87°	82	86	44°	3°	21	4	245°	214°
Pola.	137	126	94	75	145	115	36	26	57	48	164	135
Zi-Ka-Wei . .	203	211	110	84	31	11	73	56	112	115	99	76
Manila	128	134	112	99	31	64	199	200	79	88	186	167
Batavia. . . .	14	14	102	201	51	81	212	210	38	34	185	187
					Winter.							
Pavlovsk . . .	15	20	340	24	45	16	219	133	15	3	184	—
Pola.	29	33	20	357	19	30	90	68	30	30	129	129
Zi-Ka-Wei . .	66	75	148	122	80	82	114	128	89	96	90	67
Manila	80	80	205	184	113	115	205	247	82	92	221	198
Batavia. . . .	178	195	269	267	77	119	219	214	13	14	347	315
					Year.							
Pavlovsk . . .	53	61	92	79	18	48	35	356	16	1	220	—
Pola.	75	68	83	61	79	70	41	35	42	38	152	133
Zi-Ka-Wei . .	129	137	119	93	53	43	103	121	101	105	95	72
Manila	74	80	144	128	72	90	204	196	77	86	204	183
Batavia. . . .	82	101	267	263	64	100	216	212	13	14	193	210

The comparison may be completed by giving the results of VAN BEMMELEN'S harmonic analysis of the semi-diurnal part of the variation, so far as they are comparable with our present result. Only the annual mean values of $E_3^2(a)$, $E_3^2(b)$, $I_3^2(a)$, $I_3^2(b)$ can be compared in this way. They are as follows :—

	$E_3^2(a)$.	$E_3^2(b)$.	$I_3^2(a)$.	$I_3^2(b)$.
VAN BEMMELEN	1·2	4·2	0·1	3·5
Present paper	0·3	4·9	1·7	2·8

The values in the last line are formed from those given in Table G for the mean solstice (weight 2) and equinoxes (weight 1). The order of magnitude of the above two sets of determinations is the same, and although there are phase differences the amplitudes are closely similar.

We now pass on to consider the connection between the external and internal fields determined from the solar and lunar diurnal variations, the material being the results contained in Tables F and G.

Part IV.—The Connection Between the External and Internal Magnetic Variation Fields.

§ 15. *The Observed Values of the Amplitude Ratios and Phase Differences.*

In the present section the subject of discussion will be the relation between the external and internal magnetic variation fields, as measured by the amplitude ratio $\mathbf{E}_m^n/\mathbf{I}_m^n$ and the phase difference $\mathbf{e}_m^n - \mathbf{i}_m^n$. We shall not be concerned, for the time being, with the actual values of $\mathbf{E}_m^n$ and $\mathbf{e}_m^n$. The values of the amplitude ratios and phase differences for the solar diurnal magnetic variation are given in Table I. The values of $\mathbf{E}_m^n$, $\mathbf{I}_m^n$, $\mathbf{e}_m^n$, $\mathbf{i}_m^n$ and the amplitude ratios and phase differences for the lunar diurnal variation, calculated from Table C, are given in Table J (*cf.* the first six columns).

Table I.—Comparison of the External and Internal Solar Diurnal Magnetic Variation Fields.

n.	m.	Sunspot maximum, 1905.				Sunspot minimum, 1902.				Mean.	
		Mean equinox.		Mean solstice.		Mean equinox.		Mean solstice.			
		$\mathbf{E}_m^n/\mathbf{I}_m^n$.	$\mathbf{e}_m^n - \mathbf{i}_m^n$.	$\mathbf{E}_m^n/\mathbf{I}_m^n$.	$\mathbf{e}_m^n - \mathbf{i}_m^n$.	$\mathbf{E}_m^n/\mathbf{I}_m^n$.	$\mathbf{e}_m^n - \mathbf{i}_m^n$.	$\mathbf{E}_m^n/\mathbf{I}_m^n$.	$\mathbf{e}_m^n - \mathbf{i}_m^n$.	$\mathbf{E}_m^n/\mathbf{I}_m^n$.	$\mathbf{e}_m^n - \mathbf{i}_m^n$.
			°		°		°		°		°
1	2	2·9	− 5	2·8	− 3	2·7	−23	3·0	−20	2·8	−13
2	3	2·4	−18	2·3	−19	2·0	−17	2·2	−18	2·2	−18
3	4	2·4	−21	2·7	−20	2·5	−21	2·4	−21	2·5	−21
4	5	2·2	−23	2·3	−15	2·9	−30	3·2	−24	2·7	−23
Mean		2·5	−17	2·5	−14	2·5	−23	2·7	−21	2·55	−19

Mean of 1902 and 1905.

n.	m.	Q_n^n.				Q^n_{n+2}.					
		½ (Summer − Winter).		½ (Spring − Autumn).		½ (Summer − Winter).		½ (Spring − Winter).			
			°		°		°		°		
1	1, 3	2·5	− 7	2·1	− 1	2·7	+ 3	3·0	− 4		
2	2, 4	2·3	− 8	2·6	+ 2	2·0	−31	0·5	+78		
3	3, 5	2·0	−32	2·2	−13	1·1	+42	4·2	−15		
4	4, 6	1·7	−30	1·7	−19	3·0	+65	0·4	+12		
Mean		2·1	−19	2·2	− 8						

TABLE J.—Comparison of the External and Internal Lunar Diurnal Magnetic Variation Fields.

The unit is 10^{-7} C.G.S.

n.	m.	External.		Internal.		$\frac{E_m^n}{I_m^n}$.	$e_m^n - i_m^n$.	f' calculated.	a calculated.
		E_m^n.	e_m^n.	I_m^n.	i_m^n.				
Mean Solstice, ½ (Summer + Winter).									
			°		°		°		°
1	2	22·2	101	7·9	124	2·8	−23	2·6	−21
2	3	4·9	83	2·7	112	1·8	−29	2·5	−20
3	4	0·33	106	0·12	125	2·7	−19	2·5	−20
4	5	0·0120	121	0·0044	173	2·7	−52	2·7	−22
Equinox, Spring and Autumn.									
1	2	20·5	81	11·0	110	1·9	−29	2·6	−21
2	3	5·5	73	3·7	104	1·5	−29	2·5	−20
3	4	0·43	83	0·16	97	2·7	−14	2·5	−20
4	5	0·022	100	0·011	117	2·0	−17	2·7	−22
Solstitial Inequality, ½ (Summer − Winter).									
1	1	41·9	72	17·7	103	2·4	−31	2·8	−13
2	2	14·8	77	7·8	83	1·9	−6	2·3	−15
3	3	1·11	94	0·59	106	1·9	−12	2·3	−16
4	4	0·017	110	0·006	93	2·8	+17	2·4	−18

The numbers in Table I, relating to the solar diurnal magnetic variation, are remarkable for the almost unbroken uniformity (neglecting the uncertain values for Q^n_{n+2}) with which they indicate that the external magnetic field is about 2·5 times as great — reckoning by the surface values of the potentials—as the internal field, and that the latter is in *advance* of the former, in phase, by about 20 degrees. The differences between the values for the various harmonic terms, of different degrees and periods, are much less noteworthy than the accordance exhibited: the differences, moreover, appear to be in part real (§ 17). If the sixteen values of $\mathbf{E}_m^n/\mathbf{I}_m^n$ for the annual harmonics Q^n_{n+1} are treated as if their differences were altogether accidental, the probable error of the mean, 2·55, is found to be only 0·06. It may also be noticed that the nearly constant value 2·5 corresponds to very different values of $A_m^n/\mathbf{A}_m^n$, $B_m^n/\mathbf{B}_m^n$, for the various values of m, and that these different ratios are actually observed.

The general result that the external field is about $2\frac{1}{2}$ times as great as the internal field, at the earth's surface, lies between the conclusions of SCHUSTER ($\mathbf{E}_m^n/\mathbf{I}_m^n = 4$, approximately) and FRITSCHE ($\mathbf{E}_m^n/\mathbf{I}_m^n = 1·5$, approximately); VAN BEMMELEN obtained still lower values, both for the solar and lunar semi-diurnal variations. Hitherto the evidence afforded by the third and fourth harmonics has never been examined. There seems now no reason to doubt that the internal field is merely an induction product of the external field.

If the latter be so, the mechanism by means of which the internal solar variation field is produced must also be that responsible for the internal lunar variation field, and the relation between the external and internal fields will be very similar in the two cases. Table J does, indeed, show results very similar, in general, to those of Table I, especially when the small magnitude and accidental error of the determined lunar variation are considered. The mean amplitude ratio of the external and internal fields in the lunar case is 2·3, while the mean of the phase differences (all of which, save one, have the negative sign) is −21 degrees. The results for the mean of the corresponding solar variation fields are 2·4 and −19 degrees. There seems, therefore, no reason to question the similarity of the two phenomena in this respect, although a more precise discussion of the point, with more adequate data, would be of value.

As might be anticipated, the results of Tables I and J show little dependence on season or on solar activity. The only notable difference between 1902 and 1905 is found in connection with the diurnal "annual" harmonic Q_2^1, for which the mean phase differences are −4 degrees (1902) and −21 degrees (1905). The solstitial and equinoctial results separately indicate these divergent differences, and thus tend to establish the reality of the divergence. It remains to be seen whether other pairs of years will manifest the same result, but for the present no theoretical explanation of it will be attempted. Only the mean of the two values of $\mathrm{e}_2^1 - \mathrm{i}_2^1$ will be used, but the uncertainty of this mean should be kept in mind during the discussion.

§ 16. *The Hypothesis of a Uniformly Conducting Earth.*

In § 2 a brief account has already been given of the theory proposed by SCHUSTER to explain the results of his separation of the external and internal solar diurnal magnetic variation fields. At that time the problem was to account for the induction of an internal field of one-quarter the magnitude of the primary without the production of a phase difference. It now appears that a phase difference does exist, and it may be expected that the difficulty of explanation will be lessened. The sign of the difference agrees with that predicted by the theory of induction in a uniformly conducting sphere, as Prof. LAMB's researches* show (and this was kindly confirmed by him on enquiry). The hypothesis of induction being so far substantiated, it remains to consider the actual numerical relations between the external and internal fields; the theory can be regarded as completely satisfactory only when the same amount and distribution of conducting matter in the earth will suffice in relation to all the harmonics of the many periods and degrees concerned.

* *Cf.* the appendix to SCHUSTER's first memoir, p. 513, and also 'Phil. Trans.,' 1883, p. 526.

The simplest hypothesis, that of a uniformly conducting earth, will first be considered. LAMB'S theory enables the amplitude ratio f, and phase difference α, to be calculated for a uniformly conducting sphere of radius R and specific resistance ρ for any harmonic term Q_m^n in the potential of the external primary variation field. Tables giving equivalent results for certain values of ρ and m are to be found in SCHUSTER'S paper, but as they are insufficient for the more extensive observational data of Table I further calculations have been made which are summarized in Table K. Where the two sets of values of f and α overlap they are in agreement. The Table K gives the values of f and α corresponding to the two variables m and δ on which they depend; δ is defined by the equation

$$\delta = \frac{1}{N}\frac{8\pi^2 n\mathbf{R}^2}{\rho}, \tag{20}$$

where N, the number of seconds in a day (the period corresponding to $n = 1$), is equal to 86,400 in the case of the solar diurnal magnetic variation, and 89,500 (approximately) in the case of the lunar diurnal variations.

On the hypothesis that the whole earth is uniformly conducting, we must take $\mathbf{R} = \mathrm{R}$ (§ 9), and $2\pi\mathbf{R} = 4 \,.\, 10^9$ cm. Hence for the solar diurnal magnetic variations

$$\delta = \frac{4 \,.\, 10^{14}}{1 \cdot 08}\frac{n}{\rho}, \tag{21}$$

and for the lunar diurnal variations

$$\delta = \frac{4 \,.\, 10^{14}}{1 \cdot 12}\frac{n}{\rho}. \tag{22}$$

TABLE K.—Amplitude Ratios f and Phase Difference α between a Primary (External) and Secondary (Internal) Magnetic Field, Induced in a Sphere of Uniform Conductivity corresponding to Spherical Harmonics of Various Degrees m, and for Various Values of δ, or the Ratio Frequency/Resistivity.

δ.	$m = 1$.		$m = 2$.		$m = 3$.		$m = 4$.		$m = 5$.	
	f.	α.	f.	α.	f.	α.	f.	α.	f.	α.
		°		°		°		°		°
10	4·21	47·9	5·82	66·5	8·73	75·4	12·60	80·4	17·32	83·1
20	3·24	31·9	3·59	50·5	4·82	63·5	6·62	71·5	8·89	76·5
30	2·95	25·2	2·96	41·1	3·64	54·2	4·73	63·8	6·17	70·5
50	2·70	18·9	2·50	31·2	2·80	42·6	3·35	52·5	4·12	60·5
80	2·53	14·6	2·24	24·3	2·36	33·6	2·65	42·1	3·07	50·0
100	2·47	13·1	2·14	21·6	2·21	29·9	2·43	37·8	2·76	45·0
162	2·36	10·1	1·98	16·7	1·98	23·3	2·10	29·5	2·27	35·9
200	2·32	9·0	1·93	15·0	1·90	20·9	1·98	26·7	2·11	32·3
288	2·27	7·3	1·85	12·4	1·79	17·4	1·83	22·2	1·92	27·0
338	—	—	—	—	1·75	15·9	1·77	20·4	1·85	24·9
450	—	—	—	—	1·68	13·8	1·69	17·7	1·74	21·5
612	—	—	—	—	—	—	—	—	1·65	18·4

A brief inspection of Tables I and K suffices to show that no single value of ρ can be found for which the calculated and observed values of f and α are in agreement. Indeed, even if we leave out of account the constancy of ρ for the different harmonics, no calculated values of f and α are to be found in Table K which agree with those which are deduced from observation. Thus, considering only the mean results for the "annual" harmonics Q^n_{n+1}, we may notice the following comparative figures:—

TABLE L.—Illustrating the Failure of the Hypothesis of a Uniformly Conducting Earth.

Harmonic.	Theoretical α (corresponding to observed f).	Observed α.	Theoretical f (corresponding to observed α).	Observed f.
Q_2^1	37°	13	1·9	2·8
Q_3^2	30	18	1·8	2·2
Q_4^3	39	21	1·8	2·5
Q_5^4	43	23	1·8	2·7

These figures indicate clearly that the observed relations between the external and internal fields could not arise from a uniformly conducting earth whatever its conductivity. The observed phase differences are smaller than the amplitude ratios would suggest on this assumption. The discrepancy is in the same sense as in SCHUSTER's paper, where no observed phase difference was found; but as his determination of the amplitude ratio was also larger than those of this paper, his data indicated a more outstanding failure of the hypothesis than do our present results.

§ 17. *The Hypothesis of a Non-uniformly Conducting Earth.*

The simplest form of non-uniformly conducting earth which we can consider is that discussed by SCHUSTER in his first paper, viz., a sphere containing an inner core of one degree of conductivity and an outer concentric layer of another degree. There is observational evidence for the belief that the outer crust of the earth, down to a depth which is considerable in comparison with that of the oceans and of the surface inequalities, possesses high electrical resistance. For this reason, and because of the mathematical simplicity of the hypothesis, we shall suppose that the outer layer of the earth is an absolute non-conductor. If R_c is the radius of the inner core, and ρ its resistivity, the theory referred to in § 16 will, as before, enable us to calculate the amplitude ratio f and phase difference α between the potentials of the primary external, and induced internal, fields, at the surface of the inner core ($\mathbf{R} = R_c$). Corresponding to a harmonic of degree m, however, the earth-surface potential of the primary external field will at the surface of the inner core be reduced in the ratio $(R_c/R)^m$ (*cf.* § 9). Similarly, the amplitude of the induced field, which is $(1/f)$ times the amplitude of the inducing field at the surface of the core, will at the surface of the earth be reduced in the ratio $(R_c/R)^{m+1}$. Hence the amplitude ratio of the primary and secondary fields, at the earth's surface, will be equal to $(R/R_c)^{2m+1}f$; we shall denote this by f', so that

$$f' = (R_c/R)^{2m+1} f. \tag{23}$$

The phase differences $e_m^n - i_m^n$, on the other hand, remain invariable at all radii, so that the modified form of the theory enables us to account for larger amplitude ratios, corresponding to given phase differences, than was possible in § 16. The right half of Table L shows that this is the direction in which change is required in order to fit the observational results.

The adopted procedure was as follows. The phase differences α in Table I were taken as the more fundamental observed data, and the various corresponding values of δ/n (or $\mathbf{R}^2\rho$), appropriate to the several "annual" harmonics Q^n_{n+1}, were read off from Table K. The weighted mean of the deduced values of δ/n was then formed, less weight being given to the value corresponding to Q_2^1 than to Q_3^2, Q_4^2, Q_5^4, on account of the discrepancy between the 1902 and 1905 values of $e_2^1 - i_2^1$. The adopted value of δ/n was 125. The values of α calculated on this basis are given in Table M, both for the annual harmonics used in determining δ/n, and for the larger of the seasonal harmonics, Q_n^n. The value of ρ/R_c^2 calculated from (20), when δ/n is 125, is $7 \cdot 31 \,.\, 10^{-6}$.

Table M.—Phase Differences Corresponding to a Conducting Sphere for which $\rho/R_c^2 = 7 \cdot 31 \,.\, 10^{-6}$.

Annual harmonics.			Seasonal harmonics.		
—	Phase difference.		—	Phase difference.	
	Observed.	Calculated.		Observed.	Calculated.
	°	°		°	°
Q_2^1	(13)	18·9	Q_2^1	4	11
Q_3^2	18	18·7	Q_3^2	3	13
Q_4^3	21	19·3	Q_4^3	27	15
Q_5^4	23	20·5	Q_5^4	24	17
Mean . .	19	19·3	Mean . .	15	14

The agreement between the observed and calculated values of α in Table M is good, especially for the annual harmonics, which are better determined than are the seasonal harmonics (*cf.* Table I). The differences are in most cases easily within the limits of accidental error.

As regards the lunar diurnal magnetic variation, the corresponding value of δ/n is 121, and, as Table K shows, the above values of α would hardly be affected by the change. The calculated values for the lunar variation are given in Table J, alongside the observed data. Considering the uncertainties in the determination of the small quantities concerned, the agreement between the two sets of values is good. The observed phase differences are, with one exception, of the right sign, and their mean ($-20°\cdot6$) is in satisfactory accordance with the mean value of α ($-19°\cdot0$). The lunar variation, therefore, supports the above hypothesis as fully as the reliability of the data allows one to expect.

The theoretical values of f corresponding to the values of α in Table M were then compared with the observed amplitude-ratios (which we denote by f'), and in accordance with (23), the values of $(f'/f)^{1/2m+1}$, or R/R_c, were calculated for each of the annual harmonics Q_{n+1}^{n}. The four values found were

$$(23a) \qquad 1\cdot068, \quad 1\cdot030, \quad 1\cdot041, \quad 1\cdot041 \qquad (R/R_c).$$

Adopting the value $1\cdot04$, the following calculated values of f' were deduced from the formula

$$(24) \qquad f'_{\text{calc.}} = (1\cdot04)^{2m+1} f_{\text{calc.}}$$

both for the annual and seasonal harmonics :—

TABLE N.—Amplitude Ratios of the Surface Potentials of the External and Internal Diurnal Magnetic Variation Fields.

Annual harmonics.				Seasonal harmonics.			
—	$f_{\text{calc.}}$	f'. Calculated.	f'. Observed.	—	$f_{\text{calc.}}$	f'. Calculated.	f'. Observed.
Q_2^1	2·05	2·49	2·8	Q_1^1	2·42	2·72	2·3
Q_3^2	1·83	2·41	2·2	Q_2^2	1·88	2·29	2·45
Q_4^3	1·74	2·47	2·5	Q_3^3	1·73	2·27	2·1
Q_5^4	1·70	2·61	2·7	Q_4^4	1·67	2·37	1·7
Mean		2·50	2·55	Mean		2·41	2·14

Again the agreement with observation must be considered satisfactory, so that with the aid of only two disposable constants (ρ and R_c/R) a good account has been given of the values of sixteen observational quantities (eight values of the amplitude ratio and eight of the phase difference). Naturally, however, the hypothesis of a non-uniformly conducting earth such as we have considered must be regarded as giving only a convenient idealized representation of the real facts.

The theoretical values of f' calculated for the lunar diurnal variation are given in Table J; they differ but little from those of Table N. The observed values in Table J are somewhat irregular, but their mean (2·3) is in satisfactory agreement with the calculated mean (2·5), when the accidental error of the lunar data is considered.

§ 18. *The Electrical Conductivity of the Earth as Deduced from the Diurnal Magnetic Variations.*

In § 17 the following two quantities were determined, in connection with the theory that the earth has a conducting nucleus of radius R_c and specific resistance ρ, surrounded by a non-conducting layer:—

$$\rho/R_c^2 = 7{\cdot}31 \,.\, 10^{-6}, \qquad R/R_c = 1{\cdot}04. \tag{25}$$

Here R denotes the radius of the earth ($2\pi R = 4 \,.\, 10^9$ cm.).

The thickness of the outer layer is given by

$$R - R_c = R(1 - R_c/R) = 245 \text{ km.}, \tag{26}$$

or about 160 miles. The specific resistance ρ of the inner core is similarly found to be as follows:—

$$\rho = 7{\cdot}31 \,.\, 10^{-6} \,.\, R_c^2 = 2{\cdot}74 \,.\, 10^{12} \text{ C.G.S.} \tag{27}$$

These values may be compared with those deduced by SCHUSTER in his second memoir.* The calculation there made was intended to give only a rough estimate, and in order to explain the apparent absence of phase difference between the external and internal fields (*cf.* § 2) it was necessary to assume a high—practically infinite—conductivity of the inner core. Hence no comparison with (27) is possible. On this basis, however, the deduced value of the thickness of the non-conducting layer was 1000 km., in place of the present value 245 km. The difference seems altogether beyond the probable limit of error in the latter result, and 1000 km. must be regarded as definitely too large. The estimate 245 km. can hardly be liable to so much as 50 per cent. error, so that the outer layer is probably from 200 to 300 km., or 100 to 200 miles in depth. It should not be forgotten, however, that we have no evidence for a sharp line of demarcation between the outer non-conducting and the inner conducting matter.

* 'Phil. Trans.,' A, vol. 208, 1907, p. 169.

As regards the resistivity $2{\cdot}74 \cdot 10^{12}$, we may note that this is considerably less than that of the *dry* constituents of the outer rocky crust of the earth. Ordinary sea water is more conducting; thus while for distilled water ρ is about $1{\cdot}4 \cdot 10^{15}$, and for rain water $6 \cdot 10^{13}$, SCHMIDT* found that for North Sea salt water ρ is $2{\cdot}5 \cdot 10^{10}$, and ULLER† states that water from the Mediterranean Sea may be only two-thirds as resisting as this. ULLER also finds that the resistivity of moist earth ranges from 10^{13} to 10^{14}, but that for dry earth it is about 10^{15}. LÖWY‡ has measured the specific resistance of the ordinary constituents of the earth's crust (rock, stone, and so on) and concluded that for the majority of specimens ρ is greater than 10^{16}, though the results varied somewhat with the moisture in the stone. It would seem, therefore, that apart from the comparatively shallow oceanic or water-bearing strata at or near the earth's surface, the outer crust is from 100 to 1000 times as resisting as, from our calculations, the inner core appears to be.

The above data apply only to the solid crust, which geologists consider, on the evidence of seismological and gravity measurements and the study of radio-activity, to extend down to a depth of 30 or 40 miles only.§ Very little is known of the nature or condition of the underlying substance. The above value of ρ, $2{\cdot}74 \cdot 10^{12}$, is much greater than the resistivity of metals such as iron at ordinary temperatures (for which ρ is about 10^{7}); their resistance, however, increases with temperature, and may also be affected by the great pressures to which the interior layers of the earth must be subject.

As regards the depth of the non-conducting layer, 200 to 300 km., it may be noticed that HELMERT‖, and also TITTMANN and HAYFORD¶, have concluded that the inequalities in the distribution of mass near the earth's surface extend down to a depth of 120 km.; variations in the electrical properties of the earth seem therefore to extend below this region of variation of elasticity.

PART V.—ON CERTAIN PROPERTIES OF THE EARTH'S ATMOSPHERE.

§ 19. *The Solar Diurnal Barometric Variation.*

As was remarked in § 6, the atmospheric motions to which the daily magnetic variations are to be attributed, on the STEWART-SCHUSTER theory, are the horizontal and not the vertical movements. Before proceeding with the study of the magnetic variations, a brief statement will be made of the principal relevant facts regarding the daily circulation and electrical conductivity of the atmosphere.

* SCHMIDT, 'Jahrbuch d. Drahtlosen Telegraphie,' vol. 4, p. 636, June, 1911.

† ULLER, *ibid.*, p. 638.

‡ LÖWY, 'Ann. d. Physik,' 36, p. 125, October 3, 1911.

§ *Cf.* Sir A. GEIKIE'S Article on Geology, 'Encyc. Brit.' (11th ed.), vol. 11, p. 654.

‖ HELMERT, 'Encyc. d. Math. Wiss.,' VI., 1, B, vol. 2, 1910.

¶ "Geodetic Operations in the U.S.A., 1906–9." 'Report to 16th Conference of the International Geodetic Association,' by O. H. TITTMANN and T. HAYFORD.

As regards the former, the sources of information are the diurnal variations of barometric pressure, of air temperature, and of wind. ANGOT* and HANN† have studied the daily barometric variations in great detail, and DINES‡ has discussed the daily changes of wind at St. Helena. GOLD § has examined the theoretical relations between these phenomena and the air-temperature variations.

The 24-hour component of the barometric variation is very irregular in its distribution over the earth, varying greatly both in amplitude and phase with season, situation (continent, land or ocean, mountain or valley), and weather conditions. The 12-hour component, on the contrary, is one of the most regular of all meteorological phenomena. Its phase is almost exactly the same over the whole region between latitudes ± 60 degrees (at least), its amplitude shows a regular diminution with increasing latitude, while it is practically independent of longitude, weather conditions, and local situation. Mountain records indicate that the 24-hour component diminishes with increasing height, vanishing and re-appearing with reversed phase. The 12-hour component likewise diminishes in amplitude, but almost proportionately to the pressure,|| while its phase is gradually retarded. GOLD assigns 90 degrees as the probable total diminution in the corresponding phase.

The annual changes in the 24-hour barometric variation are not very regular; those in the 12-hour component, on the contrary, are simple and definite. The phase is constant throughout the year, while the amplitude has maxima at the equinoxes and unequal minima at the solstices, the total variation, however, being small. The solstitial minima are simultaneous in the two hemispheres, the principal minimum occurring at aphelion in June.

ANGOT has shown that there is also a harmonic of period eight hours having a regular annual variation, but it is too small to require consideration in this paper.

The dependence on latitude of the amplitude of the 24-hour component is rather uncertain; ANGOT gives the law as $\sin^2\theta$, θ being the co-latitude. SCHUSTER, in his second memoir (§ 6), used the harmonic Q_1^1 or $\sin\theta$, stating however, that the harmonic Q_3^1 might also be present. Expressed in millimetres of mercury, the value actually used was, at the equator,

$$0{\cdot}3 \sin t. \tag{28}$$

ANGOT found that the amplitude of the 12-hour component was mainly proportional to $\sin^4\theta$, but contained in addition a term proportional to $\sin^2\theta$, as ADOLF SCHMIDT

* ANGOT, 'Annales du Bureau Central Météorologique de France,' 1887, pp. 237–344.

† HANN, "Lehrbuch," and also numerous papers in the 'Met. Zeitschrift' and the publications of the Vienna Academy.

‡ DINES, 'Meteorological Office Publication No. 203,' 1910.

§ GOLD, 'Phil. Mag.,' 19, p. 26, 1910.

|| The diminution of amplitude seems to be slightly more rapid for the 12-hour amplitude than for the total pressure.

has also remarked; GOLD has used the law $\sin^3\theta$, which fits the observations very closely. SCHUSTER adopted the law $\sin^2\theta$ (or Q_2^2), which represents the facts moderately well, though distinctly less well than $\sin^3\theta$. SCHMIDT's expression was

(29) $$(0{\cdot}31\,Q_2^2 - 0{\cdot}082\,Q_4^2)\sin(2t+154^\circ).$$

The seasonal variation of amplitude has been represented by ANGOT by the formula

(30) $$\cos^2\delta/d^2,$$

where δ is the sun's declination and d its distance. Its magnitude is well illustrated by the following results of an analysis of the Batavian barometric observations for the period 1866 to 1905 :—

Spring (February to April)	$1{\cdot}026\sin(2t+156^\circ{\cdot}0)$,
Autumn (August to October) . . .	$1{\cdot}022\sin(2t+163^\circ{\cdot}9)$,
Summer (May to July).	$0{\cdot}935\sin(2t+158^\circ{\cdot}5)$,
Winter (November to January). . .	$1{\cdot}009\sin(2t+161^\circ{\cdot}9)$,
Mean equinox.	$1{\cdot}021\sin(2t+159^\circ{\cdot}9)$,
Mean solstice	$0{\cdot}971\sin(2t+160^\circ{\cdot}3)$.

The mode of origin of the daily barometric variation has been much discussed, but the question whether the important semi-diurnal component is of tidal or thermal origin, or both, seems still open. If it is fundamentally a tidal effect, resonance with a free atmospheric period of 12 hours must be assumed, since the lunar diurnal barometric variation (which can hardly be of other than tidal origin) is of much smaller magnitude. Probably resonance is necessarily involved also if the cause is thermal, as the KELVIN-MARGULES theory supposes. In any case, however, the 12-hour variation is clearly much more fundamental than the 24-hour component, a fact which has an interesting bearing on the magnetic variations.

If Φ is the velocity potential of the atmospheric motion, so that $\frac{\partial\Phi}{\partial s}$ is the velocity in the direction of ds, the simplest theory connecting ψ and the pressure variation δp asserts that

(31) $$\frac{1}{v^2}\frac{d\Phi}{dt} = -\frac{\delta p}{p},$$

where v is the velocity of sound. At the earth's surface, taking δp as

(32) $$0{\cdot}3Q_1^1\sin(\lambda+t') + (0{\cdot}31Q_2^2 - 0{\cdot}082Q_4^2)\sin\{2(\lambda+t')+154^\circ\}$$

in millimetres of mercury (so that p in the same units is 760), we find that

(33) $$\Phi = (Nv^2/2\pi p)\left[0{\cdot}3\cos(\lambda+t') + \{0{\cdot}16Q_2^2 - 0{\cdot}041Q_4^2\}\cos\{2(\lambda+t')+154^\circ\}\right].$$

The numerical value of $Nv^2/2\pi p$ is $1\cdot 99 \,.\, 10^{10}$ ($N = 86400$, $v^2 = 11\cdot 0 \,.\, 10^8$) or $31\cdot 3R$, where R is the radius of the earth.

The calculated values of the semi-diurnal components of velocity to east and south, at the latitude of St. Helena (16° S.), are approximately given by

$$(34) \qquad \begin{cases} \text{(East)} & -21 \sin (2nt + 154^\circ) \text{ cm./sec.,} \\ \text{(South)} & 9 \sin (2nt + 244^\circ) \text{ cm./sec.} \end{cases}$$

J. S. Dines has determined the actual values at St. Helena to be

$$(35) \qquad \begin{cases} \text{(East)} & -22 \sin (2nt + 158^\circ), \\ \text{(South)} & 35 \sin (2nt + 237^\circ). \end{cases}$$

The agreement in phase is therefore very good, and also in amplitude for the easterly component; the southerly component variation is, on the contrary, much larger than the simple theory would predict.

It has already been remarked that $\delta p/p$ seems to diminish upwards, so that in accordance with (31), the value of Φ in (33) should diminish in amplitude with increasing height.* The phase should also vary with height in the same way as for the pressure variation.†

§ 20. *The Lunar Diurnal Barometric Variation.*

Laplace, in the 'Mécanique Céleste,' xiii., ch. 1, seems to have been the first to mention that tidal motions should be present in the atmosphere as well as in the oceans. He also discussed a series of barometric observations made in France and found definite evidence of a very small lunar semi-diurnal variation, which he inclined to attribute to the indirect (rather than direct) tidal action of the moon working through the lunar tidal motion of the sea. Sabine ('Phil. Trans.,' 1847) proved from the discussion of two years' barometric observations at St. Helena that the magnitude of the lunar semi-diurnal barometric variation was of the order of 0·1 mm. of mercury. The most complete determination of the effect, however, has been made at Batavia ('Observations,' 1905); as in the case of the solar diurnal variations, the

* As regards the magnitude of the diminution, some European mountain observations discussed by Hann, 'Wien. Denkschriften,' 59, 1892, may be quoted, although mountain observations may not be altogether representative of the conditions in the free atmosphere at the same height. Considering the whole amplitude of the barometric variation, reduced to sea-level according to the formula $\delta p/p$, the results from heights of $1\frac{1}{2}$ to $2\frac{1}{2}$ km. were found to be 0·28 mm., or even less, whereas the normal sea-level value in the same latitude is 0·32 mm.

† Hann (*ibid.*) shows that, at the mountain stations referred to, the phase angles range from 110 or 120 degrees to 140 degrees, in place of 154 degrees.

lunar *diurnal* term is variable and irregular, while the semi-diurnal term is constant its value (calculated from forty years' observations) being, in millimetres of mercury,

(36) $$0{\cdot}063 \sin(2t+65°).$$

WAGNER ('Göttingen Abh.,' ix., 4, 1913), has also discussed six years' hourly barometric observations at Samoa, with the aim of determining the various characteristics of the lunar semi-diurnal barometric variation. The material was insufficient for the attempted purpose of investigating the effect of season, lunar distance, declination, and phase, but the mean result for the semi-diurnal tide may be quoted, viz.,

(37) $$0{\cdot}039 \sin(2t+33°).$$

The data do not suffice to determine the dependence of phase and amplitude on latitude, but we shall assume that the phase is constant, while the amplitude is specified by the function Q_2^2. The Samoan result does not support this conclusion very strongly when compared with the Batavian determination, but the material is insufficient to enable a definite judgment to be made as yet. For the present the Batavian result will be adopted as the basis for discussion in this paper. The corresponding value of the velocity potential Φ is given by

(38) $$\Phi = 32{\cdot}4R \,.\, 0{\cdot}010 Q_2^2 \cos(2t+65°).$$

§ 21. *The Electrical Conductivity of the Upper Atmosphere.*

It has already been mentioned (§ 6) that the electrical conductivity of the upper atmosphere was discussed by SCHUSTER in his second memoir. The possibility of the production of a conducting layer such as was suggested in that discussion by the agency of ultra-violet radiation from the sun has recently been considered by SWANN,* in connection with recent physical data bearing on the problem. Assuming the ionized constituent of the atmosphere to be oxygen, it is possible to determine the rate of supply of energy of ultra-violet radiation necessary to maintain the proposed conductivity (10^{-13} C.G.S. electro-magnetic units, in a layer 300 km. thick, where the average pressure is 10^{-6} atmosphere). The ionization potential for oxygen is 9 volts, and only the radiation of wave-length less than λ 1350 is available for ionization.† Considering the solar spectrum to be that of a black body at 6000° C., it appears that $1{\cdot}6 \,.\, 10^{-5}$ of the whole solar radiation would be thus available; it is found, however, that even if the total radiation of all wave-lengths were absorbed in the act of ionization, the rate of ionization would still be only one-sixteenth of what is required. SWANN points out that the simplest method of overcoming the

* SWANN, 'Terrestrial Magnetism,' XXI., p. 1, 1916.

† HUGHES, 'Proc. Camb. Phil. Soc.,' 15, p. 483, 1910; 'Phil. Mag.,' 25, p. 685, 1913.

difficulty may be the assumption of a smaller pressure in the conducting layer. He shows, indeed, that if the variation of the quantities involved in his calculations follow the same laws at low pressures as those actually determined at ordinary pressures, the conductivity should theoretically tend to an infinite value with increase of altitude. Perhaps the inference to be drawn from this is that whatever ultra-violet light is present is absorbed only in some particular layer of the atmosphere of appropriate constitution.

SWANN does not discuss the pressures and composition actually existing in the upper atmosphere. It appears likely, however, that at about 100 km. height the atmosphere contains roughly equal proportions of hydrogen and nitrogen, with only about 2 or 3 per cent. of oxygen; the pressure is approximately $3 . 10^{-6}$ atmosphere. At 170 km. hydrogen is altogether the preponderant constituent, the only other which is at all appreciable being helium (6 per cent.); the pressure is approximately $6 . 10^{-7}$ atmosphere. Owing to the lightness of hydrogen, the pressure diminishes with height much more slowly than near the base of the stratosphere. Even at 800 km. height, where hydrogen is the sole constituent (within a small fraction of 1 per cent.), the pressure is probably 10^{-9} atmosphere.* Perhaps at such high levels as these the ultra-violet radiation ($\lambda < 1350$) of the amount considered by SWANN might be sufficient to produce the required conductivity; his calculation related to oxygen, however, and how far it would be modified in the case of hydrogen is uncertain—I am not aware of the existence of the data necessary to examine this point. But it may be doubted whether, in any case, the suggested agency can be sustained as a probable cause of the ionization. In the first place, even though the solar atmosphere should allow such short-wave radiation to escape, its intensity must be much diminished, relatively to the red end of the spectrum, by scattering, and its total energy must be much less than that appropriate to a black body spectrum at 6000° C. Moreover it seems likely, in view of the close connection between solar and magnetic activity and the auroral, that the two latter terrestrial phenomena may originate in similar regions of the atmosphere. Recent observations indicate that the level of auroræ is generally between 90 and 140 km.†; SWANN's calculation seems to preclude ultra-violet radiation as the ionizing agency in the conducting layer, if this is indeed situated at the auroral level.

Whatever the origin and situation of the conducting layer, the main cause of its ionization must be in the sun, since the magnetic data of this paper indicate a very strongly marked diurnal variation. Hence spontaneous ionization, uninfluenced by

* For these atmospheric data *cf.* JEANS' 'Dynamical Theory of Gases' (2nd ed.), p. 356. If, however, as some authorities believe, there is no appreciable amount of free hydrogen in the atmosphere, the pressure will fall off much more rapidly than is described above, and the conclusions would be modified accordingly.

† STÖRMER, 'Terrestrial Magnetism,' XX., p. 159, 1915; SWINNE, 'Phys. Zeit.,' 17, p. 529, 1916, has discussed 2500 parallax determinations of the auroræ, and finds that 2098 lie between 90 and 130 km., and 322 between 130 and 200 km.

the sun, cannot be an important factor. Some form of corpuscular emission may be supposed to be responsible. SCHUSTER, in his second memoir, showed that only very rapidly moving corpuscles could possibly be so regarded; these would act as fertilizers in the absorbing layer by producing ions through collisions with molecules. Such corpuscles might be few in number compared with the total number of ions thus liberated; but if they are supposed to be of both signs, their speed of transmission from the sun must be great in order that re-combination may not take place on the way, while, if they are of one sign only, the accumulation of charge in the earth's atmosphere may present difficulties. The hypothesis therefore stands in need of numerical examination similar to SWANN'S discussion in the case of ultra-violet radiation, but at present the necessary data for this are wanting. It is difficult to imagine further alternatives, however, and the existence of the conducting layer itself can hardly now be questioned.

It may be mentioned that, since the intensity of the ionizing agent varies as the square of the resulting conductivity, the former must be from 100 to 150 per cent. greater at times of sunspot maximum than at times of minimum, the increase in the conductivity being from 35 to 60 per cent.

The phenomena of electric wave transmission also afford evidence on the present subject, and some conclusions of ECCLES* may be mentioned. Three strata of the atmosphere are proposed, the highest one (first suggested by HEAVISIDE in 1900) being strongly and permanently conducting, while the lowest is permanently non-conducting. The middle layer, the lower surface of which was roughly estimated to be 50 miles high, is a conductor by day and a non-conductor at night, the transitional region being fairly definite. The magnetic phenomena discussed in this paper indicate a layer resembling the middle stratum in Dr. ECCLES' theory, but give no evidence of the higher, permanently conducting layer.

PART VI.—THE THEORY OF THE EXTERNAL SOLAR AND LUNAR DIURNAL MAGNETIC VARIATION FIELDS.

§ 22. *Outline of the Mathematical Theory for the General Law of Atmospheric Conductivity.*

In Parts II. and III. of this paper it has been shown that the major portion of the solar and lunar diurnal magnetic variations is due to magnetic forces which possess a potential, and are therefore attributable to electric currents. These were found to be situated mainly above the earth's surface, and in Part IV. the internal current system was shown to be probably caused through induction by the external current system. The remaining task involved in the explanation of the whole phenomenon consists, therefore, in accounting for the externally circulating system of electric

* ECCLES, 'Roy. Soc. Proc.,' A, vol. 87, p. 79, 1912.

currents. The STEWART-SCHUSTER theory of their origin will form the basis of this enquiry, and the data to be considered will be those of Table F, p. 27, for the solar diurnal magnetic variations, and Table J, p. 35, for the lunar diurnal variations. The former data are the more accurate, and are alone suitable for exact numerical comparison with the results of theoretical calculation. But it will be found that great advantage accrues from the possession of data relating to these two closely similar yet independent sets of magnetic variations.

For convenience later in the discussion it is necessary at this stage to outline the mathematical analysis of the above theory. The following investigation is a continuation of two earlier studies of the same problem by SCHUSTER* and the present writer.† It is more general than the first of these, and also embodies certain simplifications of the methods of both papers. The details of the calculations are in all cases similar, however, and will be omitted here.

We suppose that the phenomenon takes its rise in a spherical shell of mean radius r and thickness e (small compared with r). The conductivity of the air in this shell will be denoted by ρ, and we shall suppose that ρ (or ρe) is a function of ω (the zenith distance of the sun from the point considered) expressible in the most general terms as a power series in $\cos\omega$. Clearly, if δ is the declination of the sun, and θ, λ are the co-latitude and longitude of the point, we shall have

(39) $$\cos\omega = \sin\delta\cos\theta + \cos\delta\sin\theta\cos t$$

at local time t ($t = \lambda + t'$, *cf.* § 9). We suppose, therefore, that

(40) $$\rho e = \text{K} \sum_{s=0}^{\infty} a_s \cos^s \omega. \qquad (a_0 = 1)$$

Consequently

(41) $$(\rho e)^2 = \text{K}^2 \sum_{s=0}^{\infty} b_s \cos^s \omega, \qquad (b_0 = 1)$$

where

(42) $$b_s = \sum_{r=0}^{s} a_r a_{s-r}.$$

It is convenient to transform ρe and $(\rho e)^2$ into Fourier series in $\cos st$ as follows:—

(43) $$\rho e = \text{K} \sum_{-\infty}^{\infty} f_s \cos st, \qquad (\rho e)^2 = \text{K}^2 \sum_{-\infty}^{\infty} g_s \cos st.$$

Here

(44) $$f_s = f_{-s} \equiv \sum_{q=0}^{\infty} {}_{s+2q}\text{C}_q \,.\, d_{s+2q} \,.\, (\tfrac{1}{2}\cos\delta\sin\theta)^{s+2q},$$

(45) $$g_s = g_{-s} \equiv \sum_{q=0}^{\infty} {}_{s+2q}\text{C}_q \,.\, e_{s+2q} \,.\, (\tfrac{1}{2}\cos\delta\sin\theta)^{s+2q},$$

* SCHUSTER, 'Phil. Trans.,' A, vol. 208, p. 185.
† 'Phil. Trans.,' A, vol. 213, p. 288.

and

(46) $$d_r = \sum_{l=0}^{\infty} {}_{l+r}C_r \,.\, a_{l+r} \,.\, (\sin\delta\cos\theta)^l, \qquad e_r = \sum_{l=0}^{\infty} {}_{l+r}C_r \,.\, b_{l+r} \,.\, (\sin\delta\cos\theta)^l,$$

so that f_s, g_s are power series in $\sin\theta$ and $\cos\theta$.

For the present we may consider an atmospheric oscillation of the general harmonic type, for which the velocity potential is

(47) $$\Phi = K_\sigma^\tau Q_\sigma^\tau \sin(\tau t - \alpha).$$

The radial magnetic intensity of the earth's field (measured positive outwards) will be denoted by V. The components of electric force, X and Y, measured towards the south and east respectively, are given by

(48) $$X = \frac{V}{r\sin\theta}\frac{d\Phi}{d\lambda}, \qquad Y = -\frac{V}{r}\frac{d\Phi}{d\theta}.$$

If we express X and Y in the form

(49) $$X = \frac{1}{r}\frac{d\mathfrak{S}}{d\theta} + \frac{1}{r\rho e}\frac{d\mathfrak{R}}{\sin\theta\, d\lambda}, \qquad Y = \frac{d\mathfrak{S}}{r\sin\theta\, d\lambda} - \frac{1}{r\rho e}\frac{d\mathfrak{R}}{d\theta},$$

the function $\mathfrak{R}$ will be the current function of the electric currents produced by X and Y.*

In order to obtain $\mathfrak{R}$ from (49), SCHUSTER first determined $\mathfrak{S}$ by eliminating $\mathfrak{R}$, and afterwards determined $\mathfrak{R}$ by the use of $\mathfrak{S}$. In my own earlier treatment of the problem I sought to avoid the calculation of $\mathfrak{S}$ by using the resistivity in place of the conductivity, so that $1/\rho e$ was the function which was expressed in the form (40). But it is better to keep ρ as the fundamental function, and this is easily effected, and $\mathfrak{R}$ directly determined, by the use of the following method.

On eliminating $\mathfrak{S}$ and multiplying both sides of the resulting equations by $(\rho e/\sin\theta)^2$—this being the step which yields the improvement of method—we obtain the following result:—

(50) $$\frac{(\rho e)^2 r}{\sin\theta}\left\{\frac{dX}{d\lambda} - \frac{d}{d\theta}(Y\sin\theta)\right\} = \frac{\rho e}{\sin^2\theta}\left\{\frac{d^2\mathfrak{R}}{d\lambda^2} + \sin\theta\frac{d}{d\theta}\sin\theta\frac{d\mathfrak{R}}{d\theta}\right\} - \left\{\frac{1}{\sin^2\theta}\frac{d\mathfrak{R}}{d\lambda}\frac{d\rho e}{d\lambda} + \frac{d\mathfrak{R}}{d\theta}\frac{d\rho e}{d\theta}\right\}.$$

* Here we neglect the effect of self-induction for the present (see, however, § 26). We define $\mathfrak{R}$ by the property that the flow across an element of length ds, measured from left to right, is $\frac{d\mathfrak{R}}{ds}ds$.

We may suppose that the solution $\mathfrak{R}$ is expressible in the form of a series of spherical harmonic functions, thus

$$(51)\qquad \mathfrak{R} \equiv K_\sigma^\tau CK \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} p_m{}^n Q_m{}^n \sin(nt-\alpha_n).$$

In this equation $Q_m{}^{-n}$, when n is positive, will be defined as equal to $Q_m{}^n$. When m is numerically less than n, $Q_m{}^n$ is zero.

When the above values of $\mathfrak{R}$ and ρe are substituted on the one side, and of X and Y on the other, (50) becomes

$$(52)\qquad \sum_{s=-\infty}^{\infty} g_s \left[\left\{\frac{dV}{d\theta}\frac{dQ_\sigma{}^\tau}{d\theta} - \sigma(\sigma+1) VQ_\sigma{}^\tau\right\} \sin(\overline{\tau+st}-\alpha) + \frac{\tau Q_\sigma{}^\tau}{\sin^2\theta}\frac{dV}{d\lambda}\cos(\overline{\tau+st}-\alpha)\right]$$

$$= -C \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} p_m{}^n R_m{}^n(s) \sin\{(n+s)t-\alpha_n\},$$

where

$$(53)\qquad R_m{}^n(s) \equiv \{m(m+1) - ns/\sin^2\theta\} Q_m{}^n f_s + f_s' \frac{dQ_m{}^n}{d\theta}$$

and

$$(54)\qquad f_s' \equiv \frac{df_s}{d\theta}.$$

By equating corresponding periodic functions of t and θ on the two sides of (52), we may determine the values of $p_m{}^n$ and α_n.

For the time being we will now limit the problem to the determination of the part of $\mathfrak{R}$ which depends solely on local time, *i.e.*, to the case in which α_n is independent of λ, so that on the left-hand side of (52) the term $dV/d\lambda$ will be omitted. This is the same thing as omitting from V the part which depends on λ. If we regard the earth as a uniformly magnetized sphere, with its magnetic axis inclined at an angle ϕ to the geographical axis, we may write

$$(55)\qquad V = C\cos\theta + C\tan\phi\sin\theta\cos\lambda_0,$$

where C is a constant (approximately equal to $-\frac{2}{3}$, having regard to our conventions of sign) while λ_0 is the longitude measured from the meridian (68° West of Greenwich) which contains the earth's north magnetic pole. The constant C has, for convenience, been already introduced in (51).

Neglecting, therefore, for the present, the second term in V, (45) becomes

$$(56)\qquad \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} p_m{}^n R_m{}^n(s) \sin\{(n+s)t-\alpha_n\}$$

$$= \frac{1}{2\sigma+1}\{\sigma(\sigma+2)(\sigma-\tau+1)Q^\tau_{\sigma+1} + (\sigma^2-1)(\sigma+\tau)Q^\tau_{\sigma-1}\} \sum_{s=-\infty}^{\infty} g_s \sin\{(s+\tau)-\alpha\}.$$

Hence it appears that for all values of n,

$$\alpha_n = \alpha. \tag{57}$$

and on equating the factors of corresponding periodic terms on the two sides of the equation (56), we find that

$$\frac{1}{2\sigma+1}\{\sigma(\sigma+2)(\sigma-\tau+1)\,Q^{\tau}_{\sigma+1}+(\sigma^2-1)(\sigma+\tau)\,Q^{\tau}_{\sigma-1}\}\,g_s = \sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty} p_m{}^n R_m{}^n(s') \tag{58}$$

where

$$s' = s+\tau-n. \tag{59}$$

There are an infinity of equations of type (58), one for each positive and negative integral value of s. Both sides of (58) may be expressed as the sum of a series of spherical harmonics of type $Q_r{}^{s+\tau}$, where (*cf.* 44, 45) r may take all integral values. By equating the factors of corresponding harmonics on the two sides, a doubly infinite set of equations is obtained, from which the doubly infinite set of constants $p_m{}^n$ may be determined.

If the atmospheric conductivity is uniform, f_s and g_s are zero except when $s = 0$, so that only the central equation of the set (58) appears, and on the right-hand side $R_m{}^n(s')$ vanishes except when $s' = 0$, *i.e.*, when $n = \tau$. Also $R_m{}^{\tau}(0) = m(m+1)\,Q_m{}^{\tau}$, and $g_0 = 1$. Thus, comparing the two sides of the equation, the only two values of m for which $p_m{}^n$ is not zero are $\sigma\pm 1$. Consequently, *a term* $Q_\sigma{}^{\tau}$ *in the velocity potential of the atmospheric oscillation produces harmonics of types* $Q^{\tau}_{\sigma+1}$ *and* $Q^{\tau}_{\sigma-1}$, *and no others, in the electric current function* $\mathfrak{R}$, *when the conductivity is uniform.* If also $\sigma = \tau$, as in the atmospheric oscillations $Q_1{}^1$ and $Q_2{}^2$, $Q^{\tau}_{\sigma-1}$ is zero and only the single harmonic $Q^{\tau}_{\tau+1}$ will appear in $\mathfrak{R}$. In this case the value of $p^{\tau}_{\tau+1}$ is found to be

$$\tau/(\tau+1)(2\tau+1). \tag{60}$$

If the atmospheric conductivity, or ρe, is of the form $K(1+a_1\cos\omega)$, s in (58) can take five values (-2 to $+2$), while s' on the right can take three values (-1 to $+1$). Hence n may range from $\tau-3$ to $\tau+3$, by (59), while m may range from $\sigma-3$ to $\sigma+3$. Only $p^{\tau}_{\sigma+1}$ and $p^{\tau}_{\sigma-1}$ contain a_0, as before, and they also contain only even powers of a_1; $p^{\tau}_{\sigma\pm1}$, $p_{\sigma\pm1}{}^{\tau\pm1}$ contain a_1 to the first and odd powers, and so on. The coefficients $p_m{}^n$ have been worked out by SCHUSTER, for this law of conductivity, to the fourth power of a_1, for the two harmonics $Q_1{}^1$ and $Q_2{}^2$ in the velocity potential. The more important of these coefficients, which will be required in the subsequent discussion, may be obtained from Table O by writing $a_2 = 0$.

For more complicated forms of ρe the calculation of the values of $p_m{}^n$, although straightforward and simple in principle, becomes increasingly laborious. In my previous investigation the calculation was carried as far as a_1^2 and a_2 for the law $K(a_0+a_1 \cos\omega+a_2\cos^2\omega)$, but this degree of approximation proves to be insufficient for the purposes of this paper. The terms have therefore been computed as far as the fourth order, for the same law and for the atmospheric oscillation Q_2^2. In this case (58) takes the form

(61) $$24g_s \sin^2\theta\cos\theta = \Sigma\,\Sigma\, p_m^n R_m^n(s').$$

The values of the more important coefficients p_m^n calculated from this equation are given in the following table :—

TABLE O.

Annual terms—

$$p_2^1 = \frac{16}{63}a_1\cos\delta + \frac{1}{1134}a_1\cos\delta\,(2a_1^2-9a_2)\,(5\cos^2\delta-4\sin^2\delta)$$

$$p_2^{-1} = -\frac{1}{189}a_1^3\cos^3\delta+\frac{1}{42}a_1a_2\cos^3\delta$$

$$p_3^2 = \frac{2}{15}-\frac{1}{270}a_1^2+\frac{2}{45}a_2-\frac{1}{24\,.\,81}a_1^4\left(\frac{53}{32}\cos^4\delta+\frac{1}{20}\sin^2\delta\cos^2\delta+\frac{11}{5}\sin^4\delta\right)$$
$$+\frac{1}{16\,.\,810}a_1^2a_2\,(59\cos^4\delta-8\sin^2\delta\cos^2\delta+80\sin^4\delta)$$
$$-\frac{1}{2700}a_2^2\,(11\cos^4\delta-8\sin^2\delta\cos^2\delta+16\sin^4\delta)$$

$$p_3^{-2} = \frac{29}{103{,}680}a_1^4\cos^4\delta+\frac{1}{540}a_2^2\cos^4\delta$$

$$p_4^3 = \frac{1}{140}a_1\cos\delta+\frac{1}{2100\,.\,240}a_1\cos\delta\,(15a_1^2-64a_2)\,(4\cos^2\delta-\sin^2\delta)$$

$$p_4^{-3} = 0$$

$$p_5^4 = \frac{1}{3150}a_2\cos^2\delta+\frac{1}{600\,.\,9450}(10a_1^4-51a_1^2a_2+40a_2^2)\cos^2\delta\,(3\sin^2\delta-2\cos^2\delta)$$

$$p_5^{-4} = 0$$

Seasonal terms—

$$p_1^1 = \frac{2}{105}a_2\sin\delta\cos\delta-\frac{1}{210}a_1^4\sin\delta\cos^3\delta$$
$$-\frac{1}{24}a_1^2a_2\sin\delta\cos\delta\left(\frac{4}{945}-\frac{1057}{45\,.\,1024}\cos^2\delta\right)$$
$$+\frac{1}{2520}a_2^2\sin\delta\cos\delta\left(\frac{26}{3}-\frac{83}{4}\cos^2\delta\right)$$

TABLE O (continued).

Seasonal Terms (continued)—

$$p_1^{-1} = \frac{1}{2100}(30a_1^4 - 160a_1^2a_2 - 744a_2^2)\sin\delta\cos^3\delta$$

$$p_3^1 = \frac{1}{360}(5a_1^2+24a_2)\sin\delta\cos\delta - \frac{1}{2160}a_1^2a_2\sin\delta\cos\delta\,(13\cos^2\delta - 43\sin^2\delta)$$
$$+\frac{1}{51,840}a_1^4\sin\delta\cos\delta\,(191\sin^2\delta - 41\cos^2\delta)$$
$$-\frac{1}{270}a_2^2\sin\delta\cos\delta\,(27\cos^2\delta + 8\sin^2\delta)$$

$$p_3^{-1} = \frac{1}{2700\,.\,48}(145a_1^4 - 690a_1^2a_2 + 48\,.\,83a_2^2)\sin\delta\cos^3\delta$$

$$p_2^2 = \frac{8}{63}a_1\sin\delta + \frac{1}{1134}a_1\sin\delta\,(2a_1^2 - 9a_2)(2\sin^2\delta - \cos^2\delta)$$

$$p_2^{-2} = 0$$

$$p_4^2 = \frac{1}{35}a_1\sin\delta + \frac{1}{252,000}a_1\sin\delta\,(15a_1^2 - 64a_2)(8\sin^2\delta + 3\cos^2\delta)$$

$$p_4^{-2} = 0$$

$$p_3^3 = \frac{1}{2160}(5a_1^2+24a_2)\sin\delta\cos\delta + \frac{1}{311,040}a_1^4\sin\delta\cos\delta\,(133\cos^2\delta - 41\sin^2\delta)$$
$$+\frac{1}{12,960}a_1^2a_2\sin\delta\cos\delta\,(13\sin^2\delta - 29\cos^2\delta)$$
$$+\frac{1}{540}a_2^2\sin\delta\cos\delta\,(\cos^2\delta - \sin^2\delta)$$

$$p_4^4 = -\frac{1}{13,440}a_1^3\sin\delta\cos^2\delta + \frac{1}{3150}a_1a_2\sin\delta\cos^2\delta.$$

We may now also take into account the second term in V, depending on longitude (*cf.* 55). On substituting this term in place of V in (52), and taking $\sigma = \tau = 2$ (so that we consider only a semi-diurnal atmospheric oscillation of type Q_2^2), the left-hand side is found to be

$$(62)\quad \tfrac{2}{5}K_2^2KC\tan\phi\sum_{s=-\infty}^{\infty} g_s[(9Q_1^1 - 4Q_3^1)\sin\{(s+2)t - \lambda_0 - \alpha\} + 2Q_3^3\sin\{(s+2)t + \lambda_0 - \alpha\}].$$

When the conductivity is uniform it follows that the corresponding parts of $\mathfrak{R}$, the current function, are as follows :—

$$(63)\qquad \tfrac{1}{15}K_2^2KC\tan\phi\,[(27Q_1^1 - 2Q_3^1)\sin(2t - \lambda_0 - \alpha) + Q_3^3\sin(2t + \lambda_0 - \alpha)].$$

The general form of $\mathfrak{R}$ may be written thus

$$(64)\quad 6K_2^2KC\tan\phi \sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\left[q_m^n \sin\{(n-1)t+\lambda_0-\alpha\}+r_m^n\sin\{(n+1)t-\lambda_0-\alpha\}\right]Q_m^n$$

in place of (51), and from (50) and (62) we obtain the following equations for q_m^n and r_m^n:—

$$(65)\qquad -g_s(\sin\theta-2\sin^3\theta) = \Sigma\Sigma r_m^n R_m^n(s'-1),$$

$$(66)\qquad 2g_s\sin^3\theta = \Sigma\Sigma q_m^n R_m^n(s'+1).$$

We shall only consider the simple law $(a_0+a_1\cos\omega)$ for the conductivity, and we shall neglect the seasonal terms containing $\sin\delta$. To the first order in a_1 the following are the values of q_m^n and r_m^n for the principal 24- and 12-hour longitude harmonics:—

$$(67)\quad \begin{cases} r_1^1 = \frac{3}{10}, & r_3^1 = -\frac{1}{45}, & q_3^3 = \frac{1}{90}, \\ r_2^0 = -\frac{19}{252}a_1\cos\delta, & r_4^0 = \frac{1}{70}a_1\cos\delta, & \\ q_2^2 = \frac{2}{63}a_1\cos\delta, & q_4^2 = -\frac{1}{840}a_1\cos\delta, & \end{cases}$$

The values in the first line of (67) necessarily agree with (63). This calculation is very incomplete, but it will sufficiently illustrate the discussion of the longitude terms in the magnetic variation, and until the main part of the phenomenon is better accounted for, it is hardly worth while to make a more elaborate determination of q_m^n and r_m^n.

The above investigation gives the method by which the electric current function $\mathfrak{R}$ is obtained. MAXWELL has shown that the magnetic potential corresponding to a term Q_m^n in $\mathfrak{R}$, at a radius R within the spherical current sheet, is given by

$$(68)\qquad -4\pi(m+1)R^m Q_m^n/(2m+1)r^m.$$

Since the lower limit of the current sheet is probably fifty* or more miles above the earth's surface (the radius of which we denote by R), the mean value of r may be perhaps 2 per cent. greater than R. In this case, for some of the higher harmonics in the Tables F and J, such as Q_5^4, the factor $(R/r)^m$ will not be quite negligible, and later it will be again referred to (§ 23).

We have so far assumed that τ, n, s are all integers, so that the periodicities, with regard to time, of the atmospheric oscillation and conductivity are commensurate. In the case of the lunar diurnal atmospheric oscillation and the atmospheric conductivity (which depends on solar time) this is not the case. The difficulty may be overcome in a fairly accurate way by regarding the conductivity as a function whose

* *Cf.* §§ 21, 24.

period is a lunar day, and allowing for the slight difference between this and the time period by supposing it to have a slowly varying phase. Thus if t in the above investigation represents solar time, and t_0 represents lunar time, we may write

$$t = t_0 + \nu, \qquad t_0 = t - \nu, \tag{69}$$

where ν measures the lunar phase, and increases from 0 to 2π during the interval between successive epochs of new moon. The atmospheric oscillation (47) now has the time factor $\sin(\tau t_0 - \alpha)$ in place of $\sin(\tau t - \alpha)$. Now $\tau t_0 - \alpha = \tau t - (\alpha + \tau\nu)$. Hence, if we replace α by $\alpha + \tau\nu$, the above investigation remains unchanged. The result must be interpreted in lunar time, however, so that a term $p_m{}^n Q_m{}^n \sin(nt - \alpha)$ now becomes

$$p_m{}^n Q_m{}^n \sin(nt_0 - \alpha + \overline{n - \tau}\nu). \tag{70}$$

Thus the phase of the magnetic variation of the same period ($n = \tau$) as the atmospheric oscillation remains invariable, while the phases of other components increase or decrease by whole multiples of 2π during the lunar month. This is what is actually observed in the lunar magnetic variation, and the data of Part III. have been obtained by allowing for this. It will be noticed, however, that the components for which n is negative vary very rapidly in phase (by $2(n' + \tau)\pi$ per lunar month, n' denoting the numerical value of n). These terms are included in the solar diurnal magnetic variation (where their phase is constant), but not in the lunar diurnal magnetic variation as here computed.

§ 23. *The Relative Amplitudes of the Magnetic Variations.*

The theoretical results of § 22 will now be applied to the discussion of the *relative* amplitudes of the magnetic variations, leaving the absolute amplitudes and phases to be considered later.

In the case of the lunar diurnal magnetic variation, the nature of the monthly changes of phase for the different periodic components indicates that the fundamental atmospheric oscillation here concerned is semi-diurnal. The type is not known (§ 20), but our assumption of the form $Q_2{}^2$ is perhaps not far from the truth.

According to the theory of § 22, the presence of components of other periods (with varying phases) indicates that the electrical conductivity of the atmosphere is not uniform and constant throughout the (solar) day. We will first consider what evidence is afforded, concerning the nature of the daily variation of conductivity, by the relative amplitudes of the harmonics of different periods.

The simplest law of variation for the latter is given by

$$1 + a_1 \cos \omega. \tag{71}$$

The amplitudes of the magnetic variations Q_m^n produced by an atmospheric oscillation Q_2^2, corresponding to this law of variation of conductivity, are proportional to the values of $\{m+1)/(2m+1)\} p_m^n$, where p_m^n is obtainable from Table O by writing $a_2 = 0$. For the present we shall consider only the "annual" harmonics Q^n_{n+1} at the equinoxes, so that we shall also write $\delta = 0$. Since the atmospheric conductivity cannot be negative, a_1 cannot exceed unity. Table O shows that the subsidiary (*i.e.*, non-semi-diurnal) harmonics are greater the greater the value of a_1. If we take $a_1 = 1$, the theoretical amplitudes are found to have the following relative values, k being an undetermined constant; the observed equinoctial amplitudes in the lunar diurnal magnetic variation are added for comparison (*cf.* Table J); the ratios of the two sets of numbers are also given, assuming k to be such that the ratio for Q_3^2 is unity :—

TABLE P.

	Q_2^1.	Q_3^2.	Q_4^3.	Q_5^4.
Theoretical relative amplitudes .	$15 \cdot 8k$	$7 \cdot 3k$	$0 \cdot 41k$	$-0 \cdot 00019k$
Observed amplitudes	20·5	5·5	0·43	0·022
Ratio	0·68	1·00	0·71	−0·007

The law (71) would clearly account for a considerable proportion of the harmonics Q_2^1 and Q_4^3, but wholly fails in the case of the fourth harmonic Q_5^4, both as regards magnitude and sign. This comparison shows, however, that the daily variation of electrical conductivity is at least as great as that indicated by the formula $1+\cos\omega$, since if a_1 were less than unity the amplitudes of Q_2^1 and Q_4^3 would be still smaller in comparison with Q_3^2; also it shows that the sign of a_1 must be positive, *i.e.*, that the conductivity must be great by day and small by night, since the observed phases of Q_2^1 and Q_4^3 are the same as that of Q_3^2. If a_1 were negative, their theoretical phases would be opposite to that of Q_3^2.

A more pronounced variation of the atmospheric conductivity can be represented by the law

$$(72) \qquad 1+a_1 \cos\omega + a_2 \cos^2\omega.$$

In my first paper on the lunar diurnal magnetic variation I considered the following special case of this law*

$$(73) \qquad 1+3\cos\omega+\tfrac{9}{4}\cos^2\omega = (1+\tfrac{3}{2}\cos\omega)^2,$$

which is never negative, and gives a much greater ratio of day to night conductivity than does (71). With the aid of Table O the following values of the relative

* 'Phil. Trans.,' A, vol. 213. On p. 304 a graph of this expression is given.

theoretical equinoctial amplitudes of the magnetic variations are obtained.* The ratios of these to the observed values of Table J are also given.

TABLE Q.

	Q_2^1.	Q_3^2.	Q_4^3.	Q_5^4.
Theoretical relative amplitudes	$25\cdot 3k$	$6\cdot 5k$	$0\cdot 65k$	$0\cdot 021k$
$\frac{\text{Calculated}}{\text{Observed}}$	1·0	1·0	1·3	0·8

The relative amplitudes of all four components now show very fair agreement. Clearly (72) is a great improvement on (71) as a representation of the law of variation of the electrical conductivity. Probably the true law is more complicated than (72), but we shall not trouble to seek for a closer approximation. The point gained is that, by taking a law which gives a general representation of the variation known to be probable on other physical grounds, we have been able to explain the presence and order of magnitude of magnetic variations of periods other than that of the primary atmospheric semi-diurnal oscillation.

The calculated amplitudes of the variations arising from p_2^{-1}, p_3^{-2}, and so on, comparable with those in Table Q, are found to be

$$\begin{array}{cccc} Q_2^{-1}. & Q_3^{-2}. & Q_4^{-3}. & Q_5^{-4}. \\ -4\cdot 4 & 0\cdot 8 & 0 & 0 \end{array} \qquad (74)$$

In the case of the lunar diurnal magnetic variations, however, these change their phases with great rapidity, and are not included in the "observed" amplitudes given in Table J.

We may proceed further to examine the seasonal changes in the relative amplitudes of the various components. The numbers in Table Q relate to the equinoctial variations, and are obtained by taking δ to be zero in Table O. During the solstitial quarters, however, δ is approximately 20 degrees. If this value is substituted, the following numbers, corresponding to those in Table Q, are obtained:—

$$\begin{array}{cccc} Q_2^1. & Q_3^2. & Q_4^3. & Q_5^4. \\ 24\cdot 0 & 6\cdot 5 & 0\cdot 61 & 0\cdot 018 \end{array} \qquad (75)$$

* The numbers given are $60\{(m+1)/(2m+1)\}\, p_m^n\, (R/r)^m$, the factor 60 being inserted for convenience. The factor $(R/r)^m$ allows for the fact that the magnetic variations are produced at some considerable height above the surface regions where observations are made; R/r is taken as $0\cdot 98$ (*cf.* §§ 21, 22).

Formulæ resembling those of Table O were given in my first memoir, but only carried as far as a_1^2 and a_2; in this paper some small corrections are made, and a further approximation is made by including the terms a_1^3, a_1a_2, a_1^4, $a_1^2a_2$, and a_2^2. With the above values of a_1 and a_2 the results seem to show satisfactory numerical convergence.

From Table J it appears that the amplitudes of the three components Q_3^2, Q_4^3, Q_5^4 are smaller at the solstices than at the equinoxes, while Q_2^1 is approximately constant. If the lunar diurnal atmospheric oscillation resembles the corresponding solar diurnal variation, the amplitude is greater at the equinoxes than at the solstices; this probably explains most of the variation in Q_3^2; our calculation just made would appear to indicate that, so far as the electrical conductivity is concerned, Q_3^2 is constant throughout the year. Besides the variation in the amplitude of the atmospheric motion Q_2^2, which should affect equally all the magnetic variations Q_2^1 to Q_5^4, it would appear that the non-semi-diurnal components should be further reduced at the solstices, because of the conductivity effect. Here the theory and observation are only in very rough agreement, since Q_2^1 shows a relative increase at the solstices, while the observed decreases of Q_4^3 and Q_5^4 are much more than the theory would predict.

Similar discrepancies are met with when we examine the "seasonal" magnetic components Q_n^n. These are represented in Table O by the terms involving odd powers of sin δ, which vanish at the equinoxes and change sign between summer and winter. The following values of their relative amplitudes are calculated in the same way as the numbers in Table Q :—

	Q_1^1.	Q_1^{-1}.	Q_2^2.	Q_2^{-2}.	Q_3^3.	Q_3^{-3}.	Q_4^4.	Q_4^{-4}.
(76)	$-4 \cdot 2$	$-24 \cdot 3$	$4 \cdot 7$	0	$0 \cdot 46$	0	$0 \cdot 0012$	0

The remarkable feature here is the excess of Q_1^{-1} over Q_1^1. The amplitude in Table J is that of Q_1^1. In any case, however, the above numbers, which should roughly agree with those given in Table J for the solstitial inequality (since the numbers in Table Q are approximately equal to the equinoctial data), are much smaller than the observed values.

Before considering this point further, we shall turn to the solar diurnal magnetic variation data of Table F. In this case we have no such certain means as before of determining the period or periods of the atmospheric oscillations to which the magnetic variations are due. We may notice, however, the similarity of phase ($\mathbf{e}_m^n$) between the four solar harmonics Q^n_{n+1} in Table F, which is at least as close as that shown in the lunar Table J. So far as this goes, there is a strong suggestion that the solar diurnal magnetic variations also arise from a single atmospheric oscillation.

For comparison of the amplitudes of the lunar and solar magnetic variations reference will be made to either the potentials derived from the horizontal force variations above (Tables C, G) or to those of the external fields $\mathbf{E}_m^n$ (Tables F, J). The discussion in Part IV. has shown that the potentials from the vertical force variations lead to fairly similar sub-divisions of A_m^n, B_m^n between the external and internal fields.

From Table C the amplitudes C^n_{n-1} may be calculated for the solar diurnal magnetic variation. The relative amplitudes are nearly the same in 1905 and 1902, as the following table indicates:—

TABLE R.—Ratios of C^n_{n+1} in 1905 and 1902.

	C_2^1.	C_3^2.	C_4^3.	C_5^4.
Equinox	1·43	1·31	1·25	1·35
Solstice	1·47	1·31	1·32	1 60

The increased amplitudes all round, in years of sunspot maximum, naturally point to a general increase in the electrical conductivity of the atmosphere, with little or no change in its functional dependence on the sun's zenith distance.

The lunar magnetic data refer to the quieter years of a solar cycle, and may be compared with the solar data for a mean of 1905 and 1902, giving double weight to the latter year. The comparison leads to the following results (the tenfold unit in Table F, as contrasted with Table J, should be remembered):—

TABLE S.—Ratios of Solar and Lunar $\mathbf{E}^n_{n+1}$.

	C_2^1.	C_3^2.	C_4^3.	C_5^4.
Equinox	21·8	10·4	10·0	7·6
Solstice	18·5	10·2	10·0	7·3

The ratios for the last three harmonics are in moderate agreement, especially in view of the considerable seasonal variations in the amplitudes of the third and fourth harmonics. If the ratios for the first harmonic had also been about 10, little doubt could remain that all four are produced by a single semi-diurnal atmospheric oscillation, roughly of type Q_2^2, as in the case of the lunar diurnal magnetic variations.

Even as it is, the fact that the harmonic Q_2^{-1} will appear with Q_2^1 in the solar, but not in the lunar data, leaves a possibility of explaining the above figures without introducing a further atmospheric oscillation into the theory. The agreement of phase, and the apparently more fundamental character of the 12-hour oscillation as compared with that of the 24-hour period (at the earth's surface) favour this solution.

It is worth while, however, to examine the magnetic effects of a 24-hour atmospheric oscillation. Considering the amplitudes of the magnetic components, on the same relative scale as the results in Table Q, for 12-hour and 24-hour atmospheric movements of the same equatorial amplitudes, the following values are obtained (*cf.* Table I., p. 299, 'Phil. Trans.,' A 213) :—

		Q_2^1.	Q_3^2.	Q_4^3.	Q_5^4.
(77)	12-hour wave . . .	25	6·5	0·65	0·021
	24-hour wave . . .	23	−3·3	0·24	0

The ratios in Table S might be reproduced more closely if a 24-hour atmospheric wave of about one-third the amplitude of, and in phase with, the 12-hour wave is supposed present in the solar diurnal variations.

The seasonal changes in the amplitudes of the annual harmonics in the solar diurnal magnetic variations are similar to those in the lunar variations. The diminution at the solstices is partly explicable by the similar decrease in the semi-diurnal barometric variation (§ 19), which in each case takes place without appreciable change of phase. The further reductions in the higher magnetic harmonics, at the solstices, is to be referred to the effect of the dependence of conductivity on the solar zenith distance ω, though the law (73) has been seen to be insufficient to account altogether for the observed changes. The necessary modification of (73) would seem to be in the direction of a more rapid diminution of ρ (the conductivity) as ω increases from 0 degree to 90 degrees. This is probable on other grounds (§ 21), but the theoretical discussion of its consequences would be a very laborious task.

The same modification would also increase the *theoretical* values of the seasonal harmonics Q_n^n, which for the lunar diurnal magnetic variations were found to be three or four times too small, relatively to the annual harmonics, when compared with the observed data. But a more serious difficulty arises here. If we examine the solar diurnal seasonal harmonics it is found that they are in fair agreement with the theoretical values from (73), except in regard to phase. The seasonal harmonics in the lunar variation are, in fact, nearly thrice as great compared with the annual harmonics as in the solar variation. This is immediately evident in the initial data of this paper (*cf.* Tables III. (α) and III. (γ) with VI. (c) and VI. (d)), and the following ratios of the solar and lunar seasonal harmonics $\mathbf{E}_n^n$ show the same thing :—

$\mathbf{E}_1^1$.	$\mathbf{E}_2^2$.	$\mathbf{E}_3^3$.	$\mathbf{E}_4^4$.
7·6	3·4	4·1	(14·7)

These numbers are comparable with those of Table S ; the same preponderance in the ratio for the diurnal harmonic is seen here, but the first three of these ratios are less than half as great as those for $\mathbf{E}^n_{n+1}$.

If we suppose the electrical conductivity such that the seasonal harmonics due to an atmospheric oscillation Q_2^2 have the relative magnitudes shown in the lunar diurnal magnetic variations, the defect in the solar variations would seem to require us to assume some counter-balancing seasonal magnetic harmonics in the latter, due to atmospheric oscillations of type other than Q_2^2. These can hardly be present in the lunar diurnal atmospheric movements, though no observational evidence is available. As regards the solar diurnal motions, the semi-diurnal barometric oscillation is strikingly symmetrical about the equator throughout the whole year. The 24-hour surface variation, though less definite and well-determined, also seems free from unsymmetrical components of type such as Q_2^1. The symmetrical oscillation Q_1^1, which has been suggested as a co-factor with Q_2^2 in the production of the annual magnetic harmonics, would also contribute seasonal harmonics. The amplitudes of the first two of these, comparable with (76) as are the two sets of numbers in (77), are approximately as follows:—

Q_1^1.	Q_2^2.
18	3

The first of these would tend to neutralize the corresponding harmonic due to Q_2^2, while in the second case there would be re-inforcement. The question of the exact origin of the seasonal variations must remain unsolved for the present, both their amplitudes and their phases (in view of the negative signs prefixed before the theoretical values of Q_1^1) being difficult to explain. But as regards the presence of a 24-hour oscillation in the upper atmosphere, the calculations of § 26 indicate that there are possibilities of its local production by heating effects in the conducting layer, even if the surface variation of the same period does not persist into the upper atmosphere.

§ 24. *The Absolute Values of the Amplitudes and of the Electrical Conductivity in the Upper Atmosphere.*

We have provisionally concluded that both the solar and lunar diurnal magnetic variations are, in the main, due to semi-diurnal atmospheric oscillations, roughly of type Q_2^2, in conjunction with a variable electrical conductivity which may be approximately represented by the formula (73). We will therefore now confine ourselves to the principal magnetic harmonics, Q_3^2, in discussing the absolute magnitudes of the several variables involved in the theory.

It is clear that if, as we suppose, the solar and lunar magnetic variations are similarly produced, the ratio of the amplitudes of Q_3^2 in the two cases will be equal to the ratio of the amplitudes of the corresponding semi-diurnal atmospheric oscillations. The former ratio was found in § 23 to be about 10. This is smaller than the ratio of the solar and lunar semi-diurnal barometric variations which, at Batavia for instance, is (1·00/0·063), or approximately, 16. It has already been noted, however, that (writing δp for the pressure variation at a height where the pressure is p) $\delta p/p$ diminishes somewhat with height (§ 19); HANN has explained such changes as due to the temperature variations in the lower regions of the atmosphere. As regards the lunar day, regular temperature variations should be almost or quite non-existent, and $\delta p/p$ should not alter with height. The relative decrease of the solar as compared with the lunar semi-diurnal atmospheric oscillation, from a ratio of 16 to one of about 10, may possibly be explained in this way.

In order to obtain a numerical estimate of the electrical conductivity of the region in which the magnetic variations are produced, we will determine the constant K in the formula (40) by a comparison of the lunar diurnal atmospheric velocity potential (38) with the observed magnetic variation. Considering the equinoctial "annual" harmonic Q_3^2, from Table J, we find the amplitude (*cf.* 17) in C.G.S. units to be

$$(78) \qquad 5{\cdot}5 \,.\, 10^{-7}\,\mathrm{R}.$$

The theoretical value (§ 23) is

$$(79) \qquad 4\pi \frac{m+1}{2m+1}\left(\frac{\mathrm{R}}{r}\right)^m \mathrm{K}_2^2\mathrm{C}\mathrm{K}p_3^2$$

(where $m = 3$), and, paying no attention to signs for the present,

$$(80) \qquad \mathrm{C} = \tfrac{2}{3}, \qquad \mathrm{K}_2^2 = 32{\cdot}4\mathrm{R} \,.\, 0{\cdot}010.$$

Substituting these values in (79), and equating the result to (78), we find that

$$(81) \qquad \mathrm{K} = 1{\cdot}92 \,.\, 10^{-6}.$$

Hence, approximately,

$$(82) \qquad \rho e = 2 \,.\, 10^{-6}(1+3\cos\omega+\tfrac{9}{4}\cos^2\omega).$$

At points directly beneath the sun ($\omega = 0$) the value of ρe thus given is $12 \,.\, 10^{-6}$. This calculation applies to years of low solar activity. At times of solar maximum (*cf.* Table R, p. 60) ρe would rise to $17 \,.\, 10^{-6}$ or $20 \,.\, 10^{-6}$. Moreover, as we shall see when we come to take self-induction into account (§ 26), all these values must be increased by 30 or 40 per cent., and the probable maximum value of ρe which has to be explained in any theory of the conducting layer must be, approximately,

$$(83) \qquad 25 \,.\, 10^{-6}.$$

SCHUSTER's approximate determination of ρe was $3 \,.\, 10^{-6}$.* The larger value here obtained accentuates the difficulties in the explanation of the conducting layer which have been mentioned in § 21; but there seems no reason to suppose that they are insuperable.

* SCHUSTER, 'Phil. Trans.,' A, vol. 208, p. 181.

§ 25. *The Heating Effects of the Upper Air Currents.*

In his second memoir (p. 185) SCHUSTER remarked that a further consequence of the theory outlined for the magnetic variations would be the production of a sensible heating effect by the electric currents circulating in the low pressure conducting layer. This, it was suggested, might assist in the explanation of the isothermal layer of the atmosphere. The primary cause of the approximate constancy of temperature in the stratosphere is now well understood, and the suggested heating effect can have only a secondary influence on the phenomenon. For other reasons, however, it seems desirable to examine numerically this thermal consequence of the theory. The results prove to be of some interest, and may explain part of the difference between the solar and lunar diurnal magnetic variations.

If the conductivity of the upper atmosphere is small during the night hours, the electric currents in question will flow entirely or mainly in the sunlit hemisphere. The heating effect is proportional to the square of the current, so that all the harmonics in the current function, whatever their period, contribute to the heating of this one hemisphere. As the earth revolves, the temperature of a given portion of the conducting layer will begin to increase at sunrise, and the increase will continue till the time of sunset. During the night hours cooling, mainly by radiation, must take place in order that the average state may remain steady. The conducting layer will thus suffer a diurnal change of temperature in which the 24-hour term is of much greater magnitude than any sub-component. This variation, moreover, is purely solar diurnal, including even the part due to the currents which produce the lunar diurnal magnetic variations. The temperature variation will be confined mainly to the conducting layer so far as conduction and convection are concerned (the kinematic viscosity will be very high in regions of such low pressure). A corresponding variation of pressure and of motion, not confined to the conducting layer, will result, and this may possibly account for the diurnal oscillation suggested by the magnetic variations. The phase should be constant throughout the year, though the amplitude would be expected to show a seasonal change. The most important question regarding these effects is that of their absolute magnitude, however, and we proceed to a rough numerical calculation with this in view.

For simplicity the solar diurnal magnetic variations will alone be considered, though the lunar variations will slightly increase the heating effect, to a degree depending very little on lunar phase.

The principal terms in the solar diurnal magnetic variation potential at the equinoxes (mean of 1905 and 1902) are as follows (*cf.* Table F):—

$$(84) \qquad 10^6 \cdot \frac{\psi}{R} = -38Q_2^1\cos(t+33^\circ) - 5{\cdot}0Q_3^2\cos(2t+26^\circ) - 0{\cdot}39Q_4^3\cos(3t+40^\circ).$$

Only the external variation field is considered here, of course. The corresponding terms in the current function $\mathfrak{R}$ are consequently (*cf.* 68) given by

$$(85)\qquad \frac{4\pi \cdot 10^6}{\mathrm{R}}\mathfrak{R} = 63\mathrm{Q}_2{}^1 \cos(t+33^\circ)+8{\cdot}7\mathrm{Q}_3{}^2 \cos(2t+26^\circ)+0{\cdot}70\mathrm{Q}_4{}^3 \cos(3t+40^\circ)$$
$$= 189 \sin\theta \cos\theta \cos(t+33)+130 \sin^2\theta \cos\theta \cos(2t+26^\circ)$$
$$+74 \sin^3\theta \cos\theta \cos(3t+40^\circ)$$

neglecting the factors $(\mathrm{R}/r)^m$, for simplicity.

The energy expended per second in overcoming the resistance to current flow, in the conducting layer of thickness e, as given by

$$(86)\qquad \iint \frac{1}{\rho e}\left\{\left(\frac{\partial \mathfrak{R}}{\mathrm{R}\, d\theta}\right)^2+\left(\frac{\partial \mathfrak{R}}{\mathrm{R} \sin\theta\, \partial\lambda}\right)^2\right\} \mathrm{R}^2 \sin\theta\, d\theta\, d\lambda.$$

In order to avoid excessive calculation, several approximations will be made. The current function (85) is such that the currents in the dark hemisphere are small, the various harmonics largely neutralizing one another there, and reinforcing one another in the sun-lit hemisphere. We shall imagine the three terms of (85) combined into one, however, which we shall suppose confined to one hemisphere. Since the sum of their squares is less than the square of their sum, we shall represent $\mathfrak{R}$, for this purpose, by the approximation

$$(87)\qquad \frac{200\mathrm{R}}{4\pi \cdot 10^6} \sin^2\theta \cos\theta \cos 2t.$$

Also instead of the variable factor $1/\rho e$ we shall use the constant approximation $1/(\rho e)_m$, where $(\rho e)_m$ is an average value of (82) over the daylight hemisphere. The adopted value of $(\rho e)_m$ will be* $8 \cdot 10^{-6}$ or, allowing for the correction due to self-induction, approximately 10^{-5}.

When these values are substituted in (86), and the integration is taken over one hemisphere, the total expenditure of energy per second is found to be

$$(88)\qquad \tfrac{32}{35}\pi \left(\frac{200\mathrm{R}}{4\pi \cdot 10^6}\right)^2 \frac{1}{10^{-5}},$$

or, in watts,

$$(89)\qquad \tfrac{32}{35}\pi \frac{10^{-7}}{10^{-5}}\left(\frac{200\mathrm{R}}{4\pi \cdot 10^6}\right)^2.$$

The mean value over this hemisphere, per unit area, expressed in gramme-water-centigrade units, is consequently

$$(90)\qquad \frac{1}{4{\cdot}18}\,\frac{16}{35}\,\frac{10^{-7}}{10^{-5}}\left(\frac{200\mathrm{R}}{4\pi \cdot 10^6}\right)^2 = 2{\cdot}7 \cdot 10^{-13}.$$

* *Cf.* the table of values of $1+3\cos\omega+\frac{9}{4}\cos^2\omega$ on p. 303, 'Phil. Trans.,' **A**, vol. 213.

As the heating of any volume element proceeds continuously for twelve hours, the total thermal energy communicated during the daylight hours (and lost during the night time) is

$$(91) \qquad 2{\cdot}7 \,.\, 10^{-13} \,.\, 43{,}200 = 1{\cdot}2 \,.\, 10^{-8}.$$

This refers to one square centimetre of the conducting layer of thickness e, and it may be noticed that the calculation is independent of e and of the situation of the layer.

The pressure variation produced by this temperature change is much more uncertain, since the heating effect depends greatly on the density of the atmosphere of the conducting layer; also the variation of pressure will be less than that calculated from the equation $\partial p/p = \partial T/T$, on account of the yielding of the adjacent atmospheric layers. If we suppose that the conducting layer lies between 90 and 140 km. above the earth's surface, its mass per square centimetre column is

$$(92) \qquad 760 \,.\, 2 \,.\, 10^{-6} \,.\, 13{\cdot}6 \text{ gm.},$$

13·6 being the density of mercury, and the difference of the pressures at top and bottom being approximately $2 \,.\, 10^{-6}$ atmosphere. The specific heat of air at the temperature of the atmosphere is about 0·24 (at constant pressure), while that of hydrogen is about 3·4. For the purpose of an approximate calculation we may take the specific heat as unity. In this case the total rise of temperature which would be produced by the amount of heat (91), in the above mass of gas, provided there were no loss, would be

$$6 \,.\, 10^{-6}$$

in degrees centigrade. Hence $\partial p/p$, which must be less than $\partial T/T$, cannot be so great as $3 \,.\, 10^{-8}$. This is negligible compared with the estimated amplitude of the pressure variation due to the atmospheric oscillation Q_2^2, which in the upper air is approximately 1/760 or $1{\cdot}3 \,.\, 10^{-3}$ (assuming a surface amplitude of two millimetres of mercury, and a reduction in $\partial p/p$ of about one-half, in the conducting layer, § 24).

In order that the pressure variation due to the electric heating of the conducting layer might be comparable with that due to the main atmospheric oscillation Q_2^2, the pressure of the region in which the conducting layer is situated would have to be of the order 10^{-10} atmosphere. Assuming the existence of the hydrogen layers mentioned in § 21, this pressure would be attained only at a height of more than 800 km. The pressure is of the order required for the ionisation of the conducting layer by ultra-violet radiation, according to SWANN's calculation; but it seems more probable that the conducting layer is at a lower level.

§ 26. *Discussion of the Phases of the Magnetic Variations.*

The explanation of the phases of the magnetic variations is, perhaps, the most difficult part of the present problem. The data to be reconciled are as follows. The various annual harmonics in the solar and lunar diurnal magnetic variations are approximately in agreement amongst themselves, the time factors being approximately as below, in the two cases

(98) (Solar) $-\cos(nt+25^\circ)$,

(99) (Lunar) $-\cos(nt+78^\circ)$.

Neglecting self-induction and the small terms p_m^{-n} (§ 22), the theory of § 22 indicates that the velocity potentials of the atmospheric oscillations responsible for these magnetic variations should have the *same* phase, the negative sign in (68) and the negative sign of C (*cf.* 47, 51, 68) neutralizing one another.

If the simple relation (31) holds good between the pressure variation and atmospheric velocity potential, the time factors in the pressure variations corresponding to (98) and (99) should be

(100) (Solar) $\sin(2t-155^\circ)$,

(101) (Lunar) $\sin(2t-102^\circ)$.

The factor n is here written as 2, since the fundamental pressure changes appear to be semi-diurnal.

The observed pressure variations at the earth's surface (§§ 19, 20) have the time factors

(102) (Solar) $\sin(2t+154^\circ) = \sin(2t-206^\circ)$,

(103) (Lunar) $\sin(2t+\ 65^\circ) = \sin(2t-295^\circ)$.

There is consequently no kind of agreement between the observed and calculated pressure variations in the lunar diurnal case. In the solar case the two variations agree better, but it must be remembered that the observed phase diminishes with height to a considerable extent (90 degrees or possibly more—*cf.* (91)), so that the agreement between (100) and (102) would not hold good if the latter had represented the pressure variation as it is supposed to exist in the upper atmosphere, viz., approximately

(104) (Solar) $\sin(2t-296^\circ)$.

The correction to the calculated magnetic variations produced by a given atmospheric motion, due to self-induction, may be considered at this stage. In place of a time factor $\cos(nt+\epsilon_0)$, as given by the method of § 22, the factor $\cos(nt+\epsilon_0-\epsilon_m{}^n)$ must be used, where*

$$\tan \epsilon_m{}^n = \frac{2\pi}{86{,}400} \cdot \frac{4\pi R n \rho e}{2m+1}, \tag{105}$$

m being the degree of the harmonic with which the time factor is associated. If we take $5 \cdot 10^{-6}$ as a mean value of ρe throughout the day and night, the following values of $\epsilon_m{}^n$ are yielded by the formula (105).

$$\left\{ \begin{array}{llll} \epsilon_2{}^1 = 30^\circ, & \epsilon_3{}^2 = 40^\circ, & \epsilon_4{}^3 = 44^\circ, & \epsilon_5{}^4 = 47^\circ, \\ \epsilon_1{}^1 = 44^\circ, & \epsilon_2{}^2 = 49^\circ, & \epsilon_3{}^3 = 51^\circ, & \epsilon_4{}^4 = 52^\circ. \end{array} \right. \tag{106}$$

These figures are only rough, since the non-uniform conductivity may considerably affect the theoretical formula (105). It may be noticed that the lag of phase, owing to self-induction, increases with the frequency of the harmonic, so that self-induction cannot explain the apparent *increase* of phase with frequency which is observed in the solar diurnal magnetic variation (Table F).

Self-induction will also diminish the amplitudes of the magnetic variations by a factor $\cos \epsilon_m{}^n$. This will not greatly affect the *relative* magnitudes of the various magnetic harmonics, but it will affect our estimate of the magnitude of ρe in § 24, and of the heating effects in § 25. The calculated value of ρe would appear to be increased on this account by 30 or 40 per cent., and this, again, would slightly increase the above values of ϵ.

Taking into account the above phase changes due to self-induction, the calculated time factors in the pressure variations are modified as follows:—

(107) (Solar) $\sin(2t-115^\circ)$,

(108) (Lunar) $\sin(2t-\ \ 62^\circ)$.

When these are compared with (104) and (103), the "calculated−observed" phase differences are found to be

(109) (Solar) $+181^\circ$, (Lunar) $+233^\circ$.

These values almost suggest that a mistake in sign has crept into the calculation of the magnetic from the pressure variations, but the signs have been carefully

* *Cf.* MAXWELL'S 'Electricity and Magnetism,' II., § 672, or SCHUSTER, 'Phil. Trans.,' A, vol. 208, p. 172.

examined without detection of error, and they also agree with those in SCHUSTER's investigation. We must conclude that the connection between the pressure and magnetic variations is decidedly less simple than our theory has so far assumed.

One way in which this can easily be demonstrated may be indicated. If the phase of the solar diurnal pressure variation diminishes with height through 90 degrees, its phase will agree with that of the lunar diurnal barometric variation at the earth's surface. The former change of phase is partly due to the solar semi-diurnal temperature variation in the successive layers of air, and this portion of the change will presumably have no counterpart in the lunar barometric variation. The remaining part, if any, may be ascribed to friction, and this may also affect the lunar variation to a similar extent. On this hypothesis, the solar semi-diurnal oscillation of the atmosphere should be ahead of its lunar counterpart, while the corresponding magnetic variation *lags behind* the lunar magnetic variation by about 43 degrees. It would appear, therefore, that the solar pressure variation must diminish in phase, relatively to the lunar variation, by $(154^\circ-65)+43^\circ$, *i.e.*, through 128 degrees approximately. Part of this may be ascribed to temperature, but if any considerable portion is due to friction the lunar variation must be likewise affected to some extent, so that the balance of the above 128 degrees, after the "temperature" portion of it is subtracted, must be a differential friction effect. Therefore either the diminution of phase due to temperature or that due to pressure, or both, must be larger than is generally imagined. It may be noted that the frictional effects usually referred to in this connection are those due to skin friction along the earth's surface, or the eddy friction which has recently been brought into prominence by Major G. I. TAYLOR.* True viscosity is generally regarded as so small as to be negligible, but this will hardly be the case in regions where the density is extremely small.

The lunar diurnal pressure variation can hardly be other than of tidal origin. The difference of its observed phase from that which the equilibrium theory of the tides would predict ($\sin(2t+90^\circ)$ instead of the observed $\sin(2t+65^\circ)$) may, perhaps, be attributed to friction in the lower strata of the atmosphere. If the phase diminishes upwards to the value (108), the total actual retardation will be 127 degrees, or, measured from the theoretical tidal value, 152 degrees. It would be interesting to know whether there is any possibility of accounting for such large changes of phase by skin friction and viscosity. If not, there may be some hope of an explanation by a modification of the equation (31) connecting the pressure variation with the atmospheric motion.

Until the phases of the annual harmonics in the magnetic variations are explained, those of the seasonal harmonics are not likely to be accounted for, and they will therefore not be discussed here.

* "Eddy Motion in the Atmosphere," G. I. TAYLOR, 'Phil. Trans.,' A, vol. 215, p. 1 (1915). *Cf.* also 'Roy. Soc. Proc.,' A, vol. 92, p. 196 (1916).

§ 27. *The Residual Variations, and the Terms not Dependent Solely on Local Time.*

Our discussion of the magnetic variations has so far related entirely to the simple analytical representation of the observed data which has been described in §§ 10, 13. In this representation the only terms considered were those dependent on local time. It remains, therefore, to discuss the residuals in Tables III. and VI., and to examine how far they are to be attributed to the presence of variations not depending solely on local time (*cf.* § 22).

In order to abbreviate this investigation, certain general features exhibited by the residuals will be described without setting out the detailed figures. In the first place, the mean residuals for any of the nine groups of observatories are generally similar for the years 1905 and 1902, and, in the case of the "annual" residuals in Tables III. (α) and III. (β), for the equinoxes and solstices. They are, however, greater for 1905 than for 1902, and greater at the equinoxes than at the solstices. If all the nine group mean residuals from any Table are combined numerically (counting all signs positive), the ratio of the 1905 and 1902 sums, or of the equinoctial and solstitial sums, can readily be determined. The former ratio is rather less than those of Table R, being approximately 1·2, for the "annual" residuals. The increase, such as it is, confirms the view that the residuals are a real part of the phenomenon, and do not merely represent accidental errors of observations.

The ratio of the equinoctial to the solstitial "annual" residuals is greater, being about 1·4. This is shown in the case of all three magnetic elements, and all four periodic components, the separate mean ratios for these ($n = 1, 2, 3, 4$) being 1·4, 1·2, 1·5 and 1·6. These increases roughly correspond to those shown by the Q^n_{n+1} harmonics already discussed.

The group-mean residuals, in the mean of equinox and solstice and of 1905 and 1902, taken from Tables III. (α) and (β), are collected in Table T. The "seasonal" residuals from Tables III. (γ) and (δ) will not be considered.

The largest residuals in Table T occur in the column relating to the 24-hour component variation of North force. The corresponding residuals for the West force are small and may well represent merely local peculiarities at the various observatories. If the variations indicated by the North force residuals have a potential of simple form, this must be of type Q_m^0, since this is the only type which yields North force terms without contributing also to the West force variations. In § 22 it was shown that the inclination of the magnet to the geographical axis of the earth could give rise in the current function to the variation

$$(110) \qquad 6K_2^2CK \tan\phi \, . \, r_m^0 Q_m^0 \sin(t-\lambda_0-\alpha)$$

TABLE T.—Mean Residuals, ½ (Spring + Autumn) and ½ (Summer + Winter) combined, for the Two Years 1905 and 1902 taken together.

West.		North.		Radial.	
a.	*b.*	*a.*	*b.*	*a.*	*b.*
24-hour Component.					
−4	−19	−18	6	−42	−28
1	−17	−62	−1	−24	1
17	−2	−42	1	−7	8
6	11	−102	42	−25	−11
−18	12	−52	56	16	10
−6	12	27	21	28	2
8	−36	8	−12	−57	11
−2	−17	−10	15	−56	−20
14	36	−16	−22		
12-hour Component.					
−5	−5	11	−4	−2	6
−6	−11	−4	−8	3	3
3	−4	−20	5	1	3
15	2	−36	42	−4	−8
0	0	−2	52	2	11
12	−7	5	12	6	−21
11	−32	−22	3	−14	−14
20	−24	−32	−10	24	−3
18	21	16	−16		
8-hour Component.					
−1	1	2	2	1	2
−1	2	2	6	4	4
10	−1	17	14	4	4
2	−9	0	24	−2	0
−4	−3	7	30	−1	2
10	−10	1	1	−1	−15
6	−15	−3	7	−15	−9
16	−20	26	−14	24	4
17	−14	−16	4		
6-hour Component.					
−1	4	1	0	4	3
0	4	2	2	3	2
4	1	9	6	2	1
0	−7	7	5	−2	−1
0	−6	7	6	−1	1
0	−9	−2	0	−4	−5
−1	−5	−2	9	−5	−2
10	−7	4	−8	7	4
4	−14	−8	4		

(among others), and this is of the above type. The chief coefficient r_m^0 is r_2^0 (*cf.* (67)), which is of the order $-\frac{1}{12}a_1$. The longitude λ_0 is measured from the meridian 68° West of Greenwich, while $\tan\phi$ is approximately 0·2. Assigning to a_1 the value 3, and to α and K_2^2CK the values 250 degrees and $-33R \,.\, 10^{-7}$, deduced roughly from the mean solar semi-diurnal harmonic Q_3^2, (110) becomes

(111) $$10Q_2^0 \sin(t-\lambda_0-250^\circ) \,.\, 10^{-7}R.$$

The corresponding term in the magnetic variation is

$$-6Q_2^0 \sin(t-\lambda_0-250^\circ) \,.\, 10^{-7}R.$$

The North force variation deducible from this is

$$-9 \sin 2\theta \{\sin(\lambda_0+250^\circ) \cos t - \cos(\lambda_0+250^\circ) \sin t\},$$

the unit being 10^{-7} C.G.S. This is clearly far too small to account for the residuals referred to above, and, moreover, it is found that the signs do not agree at all consistently with those in the North force a_1 column of residuals. This is owing to the factors $\sin(\lambda_0+250^\circ)$ and $\cos(\lambda_0+250^\circ)$, which vary considerably from group to group for the observatories here dealt with. The same difficulty is met with in regard to the 12-hour North force residuals; here also, moreover, the amplitude of the "longitude" harmonic (in this case the principal one is $r_1^1Q_1^1 \sin(2t-\lambda_0-\alpha)$) is too small to explain the observed residuals.

It may be noted that the magnetic variations depending on longitude would theoretically be smaller if, as suggested in § 23, the main oscillation responsible for the magnetic variations be semi-diurnal, than on the hypothesis considered by SCHUSTER; they are therefore less likely to serve as a check on the theory.

I have tried to represent the residuals of Table T by harmonics depending on the time of some standard meridian, but with little success, and I am inclined to think that they depend on local time so far as they are not merely irregularities peculiar to particular observatories. The latter can hardly be the case with regard, at any rate, to the 24-hour North force residuals; in other cases the position is much less clear. These North force residuals seem to present a difficult problem, since they apparently cannot be represented by any simple potential function. It may be recalled that the amplitude of the semi-diurnal pressure variation seems to vary with latitude according to the law $\sin^3\theta$, instead of $\sin^2\theta$ (or Q_2^2) as we have supposed; the true law could only be represented by the introduction of other harmonics besides Q_2^2 into our theoretical calculations. Another fact worth noting is that the South component of the semi-diurnal variation of wind velocity (at St. Helena) is markedly larger than that calculated from GOLD's theory* of the wind, barometric and temperature variations, while the East component seems to be in agreement with theory. Such

* *Loc. cit. ante* (§ 19).

meridianal atmospheric motions would produce electromotive forces along circles of latitude, which, again, would give rise to North force variations. It is possible that the germ of a satisfactory account of the above residuals is to be found in these tentative suggestions. Before developing them further, however, it would be desirable to examine the magnetic data more closely. But in spite of the residuals of Table T, and the points left unsettled in the previous discussion, the present analysis has revealed some important and previously unsuspected regularities in the diurnal magnetic variations, and I hope that others will thereby be encouraged to contribute further to their elucidation.

[*Note added December 9, 1918.*—In an interesting Dissertation (Utrecht, September 22, 1917: 'K. Nederland. Met. Inst.,' De Bilt, No. 102; also, in abstract, in 'K. Ak. van Wet.,' Amsterdam, 26, pp. 293–299, 1917), published since this paper was written, Miss VAN VLEUTEN has analyzed and discussed the solar diurnal magnetic variations, in order to test the theory developed by Prof. SCHUSTER in his two memoirs. The conclusions arrived at are (*a*) that "the forces causing the diurnal variation, taken as a whole, do *not* possess a potential, although it remains always possible to deduce part of these forces from a potential," and (*b*) that "the cause of the diurnal variation certainly cannot be ascribed to nothing else but a system of currents exterior to the earth and currents within the earth induced by the former system."

These conclusions appear to rest mainly on the non-correspondence of the observed North force variations with those calculated from the simple potential representation of the West force variations (*cf.* § 9). But their physical implication is that electric currents traversing the earth's surface have an important share in producing the diurnal magnetic variations. This seems extremely improbable, and, instead of (*a*) and (*b*), the interpretation of the above fact of observation seems rather to be merely that the diurnal variations are somewhat complicated, so that their potential cannot be represented exactly by any *simple* combination of spherical harmonics. There are physical grounds for such a conclusion; in a paper recently communicated to the Cambridge Philosophical Society I have given reasons for supposing that two distinct agencies and atmospheric layers are involved in the production of the diurnal magnetic variations.

While the magnetic variation field does not give simple results on the application of spherical harmonic analysis, the latter is still the only convenient means which mathematics affords for discussing the relation between the external and internal current systems, and between the former and the atmospheric circulations indicated by the barometer. I do not think that on these points the main results obtained in the present paper are likely to be modified seriously upon further investigation, but that, especially when the parts of the diurnal variations, due to the two agencies above mentioned, are separated and independently treated, the theory will be confirmed and brought more closely into accordance with observation.]

TABLE I. (*a*) (1).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Equinoxes.

(*a*) Sunspot Maximum, 1905.

(1) The 24-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_1.		b_1.		a_1.		b_1.		a_1.		b_1.	
		Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.
I.	1	76	84	83	103	−115	−137	9	57	−13	−14	−60	−48
	2	52	62	118	111	−96	−77	64	115	12	−34	−135	−138
	3	52	40	85	112	−94	−116	−1	44	7	−1	−35	−26
II.	4	90	98	88	105	−126	−162	−15	36	40	31	−29	−17
	5	25	54	85	115	−108	−114	0	43	26	22	−12	−5
	6	72	76	88	99	−144	−171	−12	37	29	25	−32	−27
III.	7	80	80	96	108	−57	−106	−47	11	76	58	−21	−19
	8	52	91	102	124	−4	−62	−21	34	53	47	0	6
IV.	9	70	72	111	112	−89	−94	−23	68	30	28	−42	−23
	10	51	67	115	122	−99	−119	−5	60	28	15	−42	−40
V.	11	11	52	84	114	47	5	−20	23	98	103	32	16
	12	−26	19	101	124	105	59	−22	15	49	59	53	42
VI.	13	−5	57	42	61	205	169	−44	−41	64	82	12	−12
	14	−15	−3	89	133	80	14	−61	−30	57	36	−5	−7
	15												
VII.	16	11	36	−103	−49	212	209	−57	−85	−104	−75	36	49
	17	−86	−22	−25	−5	56	55	−126	−195				
VIII.	18	−94	−58	−99	−54	118	108	−42	−73	−128	−90	−18	−8
	19	−69	−26	−153	−105	70	17	11	70	−209	−154	−15	14
	20												
IX.	21	−44	−42	−125	−88	−131	−105	−45	−47				

* For an explanation of Tables I., II., III., *cf.* §§7, 8. The unit of force in these three tables is 10^{-6} C.G.S.

TABLE I. (*b*) (1).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Equinoxes.

(*b*) Sunspot Minimum, 1902.

(1) The 24-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_1.		b_1.		a_1.		b_1.		a_1.		b_1.	
		Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.
I.	1	46	76	46	77	– 70	– 93	6	23	7	1	– 14	– 15
	2	52	64	107	97	– 55	– 60	46	63				
	3	41	70	53	87	– 47	– 58	5	25	– 5	5	– 15	– 9
II.	4	57	84	46	74	– 64	– 100	– 4	19	31	29	– 12	– 7
	5	18	39	62	90	– 52	– 66	4	42	22	16	4	4
	6	47	64	48	73	– 87	– 117	4	28	24	32	– 1	– 13
III.	7	58	85	51	85	– 19	– 59	– 18	5	45	36	– 5	– 12
	8	39	78	55	88	12	– 33	– 6	30	35	35	6	12
IV.	9	45	57	66	83	– 71	– 94	36	34				
	10	43	56	68	95	– 64	– 97	18	33	32	26	– 19	– 16
V.	11	32	12	– 10	25	13	50	57	84	72	58	14	1
	12	5	21	69	85	75	29	– 3	11				
VI.	13	– 6	44	18	51	141	129	– 40	– 15	41	67	0	– 6
	14												
	15	16	60	4	27	197	175	– 19	– 26	82	82	– 36	– 39
VII.	16	– 15	50	– 82	– 48	161	144	– 46	– 65	– 67	– 55	– 27	– 10
	17												
VIII.	18												
	19	– 52	– 33	– 114	– 81	37	6	15	45	– 164	– 165	– 26	– 4
	20	– 10	– 13	– 106	– 71	– 96	– 45	0	– 22	– 6	– 9	– 1	6

TABLE I. (*a*) (2).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Equinoxes.
(*a*) Sunspot Maximum, 1905.
(2) The 12-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_2.		b_2.		a_2.		b_2.		a_2.		b_2.	
		Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.
I.	1 2 3	20 14 20	76 59 76	87 98 101	84 89 95	−66 −61 −49	−64 −51 −39	6 −19 17	40 43 56	42 −2 37	32 24 31	9 36 20	4 13 2
II.	4 5 6	35 −5 25	94 68 78	105 110 99	87 93 84	−67 −68 −77	−60 −45 −67	−11 28 −13	35 77 26	45 25 48	50 25 56	5 15 4	8 2 0
III.	7 8	32 27	87 101	112 111	95 103	−21 7	−14 0	−22 13	34 79	53 46	47 47	6 9	−6 7
IV.	9 10	28 23	96 98	100 111	85 112	−63 −71	−28 −67	32 20	103 87	38 33	51 32	−2 −6	−6 −6
V.	11 12	8 5	114 97	106 93	94 84	18 41	20 38	20 30	76 45	60 28	57 51	21 37	2 6
VI.	13 14 15	20 −13	113 60	56 91	22 94	94 31	98 19	−33 −7	−20 −11	41 13	58 3	−11 −2	−69 −25
VII.	16 17	−2 −78	72 −38	−92 −37	−86 43	96 33	88 23	−29 −62	−38 −95	−64	−45	−9	−12
VIII.	18 19 20	−85 −45	−3 −33	−96 −160	−84 −142	70 58	42 42	−2 −21	−39 27	−67 −2	−47 −13	7 −7	−7 −3
IX.	21	−26	6	−89	−70	−60	−57	1	−1				

TABLE I. (*b*) (2).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Equinoxes.

(*b*) Sunspot Minimum, 1902.

(2) The 12-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_2.		b_2.		a_2.		b_2.		a_2.		b_2.	
		Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.
I.	1	30	54	58	63	− 32	− 48	5	26	17	22	2	− 1
	2	11	78	86	46	− 49	− 31	48	59				
	3	24	52	62	67	− 17	− 24	22	40	22	20	9	3
II.	4	48	72	59	70	− 36	− 46	− 2	29	30	41	0	1
	5	5	49	65	69	− 38	− 36	38	63	12	16	14	1
	6	37	63	56	69	− 46	− 53	0	25	37	40	− 5	− 9
III.	7	48	78	66	82	− 5	− 13	− 8	23	38	25	− 1	0
	8	36	77	64	74	20	1	19	61	31	34	0	6
IV.	9	57	88	67	63	− 49	− 31	41	57				
	10	54	88	74	78	− 54	− 46	36	59	27	33	− 10	− 15
V.	11	7	5	31	65	31	81	73	65	37	43	13	3
	12	39	88	66	55	27	24	16	18				
VI.	13	19	82	24	33	76	72	− 35	− 7	27	53	− 18	− 42
	14												
	15	30	97	22	23	106	83	− 27	− 19	49	42	− 17	− 23
VII.	16	− 19	53	− 77	− 77	71	60	− 30	− 32	− 41	− 36	− 26	− 24
	17												
VIII.	18												
	19	− 46	− 7	− 136	− 111	48	44	− 20	14	10	− 7	− 8	− 14
	20	− 1	13	− 112	− 85	− 62	− 49	5	− 14	− 2	− 5	− 8	− 2

TABLE I. (a) (3).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Equinoxes.
(a) Sunspot Maximum, 1905.
(3) The 8-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_3.		b_3.		a_3.		b_3.		a_3.		b_3.	
		Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.
I.	1	20	30	47	32	−35	−18	22	38	10	11	2	1
	2	10	53	16	8	−24	−10	5	25	17	−7	−14	−23
	3	19	36	55	31	−31	−19	24	41	21	18	5	2
II.	4	39	49	56	38	−33	−8	22	40	26	27	1	1
	5	7	38	75	41	−45	−22	30	44	17	13	9	−1
	6	31	48	49	34	−28	−8	15	26	23	27	−1	−1
III.	7	39	53	67	42	−10	12	12	42	36	30	0	−5
	8	45	76	70	36	−7	6	24	69	34	32	0	−3
IV.	9	28	62	43	17	−29	19	30	57	26	31	−2	−16
	10	25	71	51	32	−32	−12	25	67	18	22	−6	−11
V.	11	26	90	82	38	−9	12	21	46	37	24	2	−12
	12	24	82	55	12	15	27	15	15	27	36	18	−17
VI.	13	48	94	34	−17	31	32	−37	−32	35	27	−30	−73
	14	−28	55	48	14	2	19	4	−3	−4	−11	−5	−13
	15												
VII.	16	−25	18	−33	−52	40	33	−24	−23	−38	−26	0	−11
	17	−32	−24	−41	17	17	10	−11	−25				
VIII.	18	−62	−3	−74	−82	24	5	5	−20	−38	−27	9	−8
	19	−38	−11	−80	−79	30	32	−19	4	25	21	34	18
	20												
IX.	21	−10	9	−50	−53	−50	−38	18	20				

TABLE I. (*b*) (3).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Equinoxes.

(*b*) Sunspot Minimum, 1902.

(3) The 8-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_3.		b_3.		a_3.		b_3.		a_3.		b_3.	
		Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.
I.	1	22	33	26	29	− 17	− 19	19	28	7	8	− 2	− 3
	2	25	46	43	9	− 5	1	31	30				
	3	25	35	32	27	− 14	− 13	25	27	18	17	3	− 2
II.	4	39	50	29	27	− 18	− 12	18	37	18	25	− 2	− 4
	5	13	33	49	36	− 32	− 14	32	37	11	11	4	− 3
	6	34	46	27	28	− 23	− 8	15	25	23	24	− 6	− 6
III.	7	44	60	36	32	− 5	7	14	32	24	17	− 11	− 5
	8	47	69	34	30	9	8	26	45	22	25	− 6	− 3
IV.	9	38	58	26	14	− 8	13	26	37				
	10	36	63	32	24	− 20	0	26	42	14	18	− 8	− 8
V.	11	− 11	5	21	42	36	35	64	66	23	20	− 4	− 11
	12	30	69	33	− 1	10	17	8	3				
VI.	13	16	80	43	5	26	30	− 28	− 20	24	33	− 28	− 52
	14												
	15	23	65	29	7	33	26	− 32	− 23	32	18	− 8	− 14
VII.	16	− 23	17	− 29	− 48	26	28	− 24	− 20	− 24	− 21	− 8	− 13
	17												
VIII.	18												
	19	− 31	− 6	− 71	− 67	24	27	− 16	1	22	26	26	11
	20	− 23	− 6	− 36	− 46	− 25	53	13	− 3	− 2	− 2	− 2	0

TABLE I. (*a*) (4).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Equinoxes.
(*a*) Sunspot Maximum, 1905.
(4) The 6-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_4.		b_4.		a_4.		b_4.		a_4.		b_4.	
		Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.
I.	1 2 3	7 9 3	12 23 1	22 9 18	16 0 9	− 14 0 − 7	− 3 − 6 3	12 − 3 14	11 2 10	11 8 8	4 4 7	2 7 5	3 11 3
II.	4 5 6	16 8 15	18 8 16	26 21 22	11 4 8	− 7 − 14 − 6	1 − 3 4	8 11 6	15 13 10	13 5 13	10 3 10	− 1 5 0	0 − 2 0
III.	7 8	18 27	22 27	26 26	8 6	− 1 6	10 9	10 16	15 24	16 15	9 9	− 4 − 2	− 7 − 6
IV.	9 10	17 13	25 24	11 12	− 9 0	− 2 − 7	23 8	12 8	14 19	6 4	4 2	− 3 − 5	− 10 − 5
V.	11 12	24 22	32 35	31 16	− 2 − 20	− 6 3	6 14	6 6	8 1	14 16	7 11	− 1 0	− 8 − 19
VI.	13 14 15	28 3	24 15	5 20	− 20 − 20	5 − 6	6 7	− 12 0	− 14 − 2	12 0	− 6 − 7	− 21 − 4	− 29 7
VII.	16 17	− 16	− 9	− 2	− 20	8	3	− 8	3	− 14	− 12	5	− 4
VIII.	18 19 20	− 26 − 11	− 6 − 8	− 19 − 18	− 29 − 27	12 5	− 1 11	− 1 − 9	− 9 − 1	− 15 11	− 10 15	6 17	− 3 18
IX.	21	− 7	4	− 16	− 18	− 10	− 14	8	8				

TABLE I. (*b*) (4).—Fourier Coefficients of the Solar Diurnal Magnetic Components at the Equinoxes.

(*b*) Sunspot Minimum, 1902.

(4) The 6-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_4.		b_4.		a_4.		b_4.		a_4.		b_4.	
		Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.	Spring.	Autumn.
I.	1	7	8	14	17	− 2	− 6	10	10	6	7	0	2
	2	4	13	14	1	2	1	15	11				
	3	6	4	12	9	− 6	− 2	9	11	7	7	2	3
II.	4	16	20	16	13	− 1	0	10	16	9	13	− 2	− 2
	5	8	6	14	6	− 8	− 4	14	11	6	2	3	0
	6	16	19	15	13	− 2	2	7	13	9	10	− 2	− 5
III.	7	21	25	17	9	3	7	11	14	11	8	− 6	− 4
	8	25	27	12	7	3	10	17	16	10	8	− 7	− 6
IV.	9	17	21	3	− 8	− 1	15	10	10				
	10	17	23	8	− 5	− 4	10	5	15	2	4	− 8	− 6
V.	11	− 9	3	6	7	26	26	23	4	9	4	− 3	− 5
	12	20	25	6	− 20	1	7	0	− 2				
VI.	13	2	26	25	− 10	2	8	− 14	− 12	13	3	− 18	− 25
	14												
	15	12	12	12	− 8	− 1	0	− 14	− 15	12	3	− 3	− 5
VII.	16	− 15	− 6	− 2	− 17	4	3	− 6	− 1	− 12	− 11	4	− 3
	17												
VIII.	18												
	19	− 10	1	− 24	− 23	5	8	− 5	− 3	5	5	17	15
	20	− 11	4	− 24	18	− 3	− 5	7	1	− 1	0	1	0

TABLE II. (*a*) (1).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Solstices.

(*a*) Sunspot Maximum, 1905.

(1) The 24-hour Component.

Group No.	Observatory No.	West. a_1. Summer.	West. a_1. Winter.	West. b_1. Summer.	West. b_1. Winter.	North. a_1. Summer.	North. a_1. Winter.	North. b_1. Summer.	North. b_1. Winter.	Radial. a_1. Summer.	Radial. a_1. Winter.	Radial. b_1. Summer.	Radial. b_1. Winter.
I.	1	68	66	185	32	−155	−25	78	−11	17	−12	−34	−45
	2	38	28	176	43	−89	−46	109	67	23	4	−106	−104
	3	76	46	185	22	−135	−17	48	−18	36	−2	−33	−31
II.	4	83	70	173	36	−149	−62	52	−25	67	11	−8	−27
	5	47	43	193	31	−118	−25	38	2	46	5	−4	−18
	6	55	58	175	38	−153	−83	76	−25	54	6	−14	−29
III.	7	86	70	168	44	−71	−45	14	−42	103	28	−15	−23
	8	64	42	171	34	−44	1	38	−41	62	20	14	−9
IV.	9	68	30	135	66	−24	−89	54	5	31	23	−29	−20
	10	58	30	148	73	−45	−99	55	−11	31	25	−28	−33
V.	11	30	3	155	38	33	26	15	−26	81	56	9	7
	12	6	−19	126	55	68	79	−1	−38	35	46	41	30
VI.	13	61	−2	93	−2	200	160	−39	−47	79	49	−6	19
	14	−1	−25	124	83	60	17	−34	−76	7	41	−23	−7
	15												
VII.	16	24	5	−13	−124	165	199	−69	−35	−57	−75	37	16
	17	10	−109	−3	−78	45	35	−97	−198				
VIII.	18	−39	−133	−7	−95	50	184	−72	−52	−59	−108	8	6
	19	−32	−62	−33	−223	20	10	61	−30	−96	−240	5	−41
	20												
IX.	21	−34	−76	−14	−187	−15	−146	−31	23				

TABLE II. (*b*) (1).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Solstices.

(*b*) Sunspot Minimum, 1902.

(1) The 24-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_1.		b_1.		a_1.		b_1.		a_1.		b_1.	
		Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.
I.	1	67	49	127	3	−114	−9	40	−6	28	−3	−18	−19
	2	64	30	144	26	−79	−13	62	24				
	3	73	35	139	1	−85	8	32	−6	35	2	−10	−17
II.	4	71	52	117	5	−104	−26	28	−12	56	5	−11	−16
	5	46	25	145	4	−74	−2	31	0	25	12	3	−5
	6	41	45	125	7	−108	−44	50	−10	62	−14	−12	−6
III.	7	69	56	120	7	−45	−19	4	−20	69	22	−6	−29
	8	63	41	128	3	−17	17	23	−20	52	8	15	−8
IV.	9	60	27	107	27	−55	−56	62	8				
	10	68	23	107	31	−34	−56	49	−6	36	16	−17	−17
V.	11	34	20	18	−17	43	−3	118	−1	51	31	−5	12
	12	21	4	94	31	67	46	3	−13				
VI.	13	41	−4	76	−28	137	109	−33	−42	61	18	1	2
	14												
	15	48	−6	69	−28	163	154	−35	−19	97	70	−25	−40
VII.	16	18	−8	−1	−107	123	127	−60	−42	−57	−56	15	−3
	17												
VIII.	18												
	19	−16	−54	−16	−167	16	21	64	−7	−101	−166	9	−44
	20	−17	−24	−20	−160	−26	−84	−35	17	−4	−9	4	−1

TABLE II. (*a*) (2).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Solstices.
(*a*) Sunspot Maximum, 1905.
(2) The 12-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_2.		b_2.		a_2.		b_2.		a_2.		b_2.	
		Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.
I.	1 2 3	80 68 61	− 14 17 − 10	109 134 126	42 51 37	− 96 − 48 − 82	− 28 − 30 − 22	35 69 44	− 1 − 9 − 2	35 29 42	21 − 9 17	12 − 7 − 1	7 8 13
II.	4 5 6	97 67 72	2 − 13 0	109 136 102	54 27 54	− 79 − 77 − 82	− 43 − 19 − 52	28 41 18	− 9 7 0	71 43 76	13 11 17	15 0 13	3 6 2
III.	7 8	100 77	7 − 4	131 138	63 47	− 10 − 7	− 33 − 13	23 69	− 15 − 2	79 56	19 15	8 15	− 5 4
IV.	9 10	95 98	27 20	102 127	91 99	9 − 40	− 78 − 82	76 83	23 13	55 42	23 24	5 − 4	1 − 13
V.	11 12	83 67	− 3 8	121 77	52 73	19 43	− 3 22	45 23	3 7	46 40	34 27	6 9	18 33
VI.	13 14 15	97 44	− 18 − 26	51 85	20 101	101 36	53 − 9	− 27 − 6	− 36 − 31	63 − 3	21 29	− 43 − 23	2 − 11
VII.	16 17	50 − 27	− 20 − 108	− 52 50	− 108 − 75	71 23	85 19	− 33 − 53	− 26 − 105	− 33	− 58	− 8	− 8
VIII.	18 19 20	− 10 20	− 136 − 70	− 45 − 76	− 52 − 219	9 36	88 61	− 39 40	53 − 56	− 38 − 37	− 55 34	4 − 16	23 1
IX.	21	1	− 85	− 19	− 83	− 26	− 50	16	51				

TABLE II. (*b*) (2).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Solstices.

(*b*) Sunspot Minimum, 1902.

(2) The 12-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_2.		b_2.		a_2.		b_2.		a_2.		b_2.	
		Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.
I.	1	69	2	80	27	−59	−15	24	0	28	6	0	2
	2	84	25	90	29	−32	−34	68	10				
	3	63	−1	92	20	−41	−3	42	3	34	8	−2	2
II.	4	86	13	77	34	−51	−29	19	−3	53	7	−1	−3
	5	65	−4	93	12	−56	−9	55	13	20	−1	1	7
	6	63	8	70	36	−59	−36	15	2	60	5	−6	−7
III.	7	88	16	91	36	1	−22	8	−9	47	16	2	−9
	8	69	8	97	28	12	−4	48	−7	38	8	7	−4
IV.	9	88	32	84	67	−14	−60	58	13				
	10	100	24	86	67	−26	−62	67	11	48	15	−16	−14
V.	11	14	4	48	24	84	−10	87	15	35	18	−2	7
	12	77	19	56	59	42	7	11	5				
VI.	13	73	−8	41	1	78	46	−21	−30	52	11	−29	−1
	14												
	15	84	−4	34	1	82	74	−18	−21	50	31	−22	−4
VII.	16	38	−17	−40	−84	55	56	−30	−30	−35	−38	−18	−9
	17												
VIII.	18												
	19	21	−56	−58	−166	41	49	46	32	−21	39	−17	3
	20	23	−18	−41	−133	−29	−68	−38	21	−2	−6	1	−4

TABLE II. (a) (3).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Solstices.

(a) Sunspot Maximum, 1905.

(3) The 8-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_3.		b_3.		a_3.		b_3.		a_3.		b_3.	
		Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.
I.	1	25	14	42	9	−14	−16	26	8	11	−3	6	5
	2	29	13	16	22	−17	−20	0	8	14	2	−6	−13
	3	7	15	42	15	−20	−16	32	5	19	2	9	−3
II.	4	39	20	39	11	−6	−19	21	21	28	6	6	−1
	5	13	6	46	33	−28	−17	32	17	12	12	6	1
	6	36	16	33	13	−10	−15	11	18	25	7	−1	−3
III.	7	49	26	46	23	21	−15	26	14	37	14	5	−1
	8	49	25	47	24	2	−15	65	10	27	9	5	1
IV.	9	50	13	27	41	24	−32	50	16	26	15	−2	−2
	10	57	10	35	43	−16	−39	63	14	19	19	−3	−7
V.	11	58	13	42	54	6	−15	36	12	18	20	−8	0
	12	53	8	7	53	8	8	13	10	24	21	−8	22
VI.	13	75	14	4	19	27	11	−15	−32	34	18	−47	−11
	14	43	−3	17	54	15	−5	8	−7	−11	10	−9	−7
	15												
VII.	16	28	−36	−29	−35	25	37	−15	−21	−6	−35	−12	2
	17	−23	−34	33	−69	14	24	−8	−39				
VIII.	18	−5	−78	−54	−29	−4	24	−17	9	−24	−23	1	18
	19	18	−62	−42	−97	23	45	24	−44	1	41	2	22
	20												
IX.	21	−4	−18	−22	−31	−26	−26	11	39				

TABLE II. (*b*) (3).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Solstices.

(*b*) Sunspot Minimum, 1902.

(3) The 8-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_3.		b_3.		a_3.		b_3.		a_3.		b_3.	
		Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.
I.	1	24	9	33	10	−13	−12	22	12	7	1	2	−2
	2	34	10	13	10	−11	−10	14	0				
	3	11	12	31	7	−13	−7	30	9	16	4	6	−3
II.	4	42	21	24	11	−3	−15	22	17	22	5	−1	−4
	5	18	9	31	17	−20	−14	36	14	5	4	3	−3
	6	34	17	18	14	−5	−13	12	16	26	7	−6	−3
III.	7	52	27	28	15	20	−14	21	12	21	10	−1	−12
	8	47	20	33	15	9	−10	47	6	18	6	1	−2
IV.	9	45	17	17	32	15	−27	42	5				
	10	58	15	18	28	−5	−30	49	6	24	10	−11	−10
V.	11	5	−3	42	22	62	−2	30	22	14	−1	−12	−11
	12	55	9	3	49	13	0	3	2				
VI.	13	69	8	7	11	20	6	−16	−21	32	11	−39	−5
	14												
	15	56	−5	5	21	23	27	−11	−26	18	15	−14	7
VII.	16	29	−21	−23	−28	24	22	−18	−23	−14	−20	−8	0
	17												
VIII.	18												
	19	21	−47	−35	−72	23	29	29	−32	4	28	0	28
	20	13	−25	−25	−53	−17	−33	−27	25	0	−3	0	−1

TABLE II. (*a*) (4) Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Solstices.
(*a*) Sunspot Maximum, 1905.
(4) The 6-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_4.		b_4.		a_4.		b_4.		a_4.		b_4.	
		Summer.	Winter.	Summer	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.
I.	1 2 3	3 5 – 8	3 0 6	9 – 7 1	14 3 13	– 7 10 6	– 2 – 4 – 6	0 – 5 – 5	7 1 7	8 2 5	1 – 3 8	2 6 1	– 4 – 4 2
II.	4 5 6	– 1 – 4 2	16 8 14	3 – 4 4	14 16 12	2 8 2	– 5 – 8 – 2	– 2 3 1	6 9 9	8 – 2 5	4 3 3	0 2 – 1	– 1 1 0
III.	7 8	3 2	19 15	0 – 1	19 14	16 4	– 3 1	4 16	9 13	5 0	7 9	3 1	– 1 0
IV.	9 10	– 1 8	16 17	– 15 – 8	14 18	17 7	– 9 – 14	11 19	4 5	2 – 1	7 7	– 2 2	– 4 – 6
V.	11 12	13 11	19 16	– 15 – 21	28 27	10 2	– 4 4	4 5	9 14	– 2 2	10 14	– 5 – 14	5 6
VI.	13 14 15	7 0	20 11	– 14 – 14	– 2 29	– 2 2	4 – 2	– 3 2	– 5 6	– 3 0	5 – 1	– 13 5	– 17 – 4
VII.	16 17	1	– 6	– 14	0	3	4	0	– 7	– 2	– 8	– 4	4
VIII.	18 19 20	– 5 16	– 16 – 15	– 34 – 25	– 1 – 14	– 2 10	9 10	– 7 8	– 4 – 19	– 14 11	1 3	– 1 15	4 6
IX.	21	1	0	– 17	– 16	– 13	– 9	7	11				

TABLE II. (*b*) (4).—Fourier Coefficients of the Solar Diurnal Magnetic Variation at the Solstices.

(*b*) Sunspot Minimum, 1902.

(4) The 6-hour Component.

Group No.	Observatory No.	West.				North.				Radial.			
		a_4.		b_4.		a_4.		b_4.		a_4.		b_4.	
		Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.
I.	1	− 1	4	9	3	− 7	− 2	5	6	4	2	− 1	− 1
	2	2	3	− 5	4	0	0	1	− 2				
	3	− 4	6	6	5	4	− 2	− 6	1	6	1	5	− 1
II.	4	4	8	5	5	1	− 3.	2	9	8	3	2	− 1
	5	− 6	7	− 1	10	4	− 7	7	8	− 1	5	1	1
	6	5	9	3	7	2	− 2	4	11	4	1	− 4	− 2
III.	7	8	12	2	8	10	− 3	5	9	2	5	2	− 5
	8	5	15	− 3	5	5	− 3	8	6	0	4	0	− 1
IV.	9	9	12	− 13	15	14	− 9	13	3				
	10	9	13	− 6	14	9	− 10	18	5	4	5	− 4	− 7
V.	11	5	0	7	13	15	5	− 12	14	− 2	4	− 2	0
	12	14	9	− 18	22	1	2	3	4				
VI.	13	15	13	− 9	0	2	3	5	− 6	0	5	− 16	− 10
	14												
	15	8	2	− 15	13	− 4	1	1	− 11	− 4	8	− 5	2
VII.	16	5	− 5	− 11	0	7	3	2	− 4	0	− 3	− 11	3
	17												
VIII.	18												
	19	19	− 10	− 18	− 9	9	9	11	− 12	2	3	10	4
	20	11	− 10	− 13	− 1	− 2	− 3	− 14	9	− 2	0	0	0

TABLE III. (α) (a).—Fourier Coefficients (Group Means) of the Solar Diurnal Magnetic Variation, at Sunspot Maximum, 1905.

(α) The Mean Equinoctial Component, $\frac{1}{2}$(Spring + Autumn).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(1) The 24-hour Component.												
I.	61	66	102	138	−106	−71	48	34	−7	55	−74	−19
II.	69	62	96	128	−138	−40	15	19	29	59	−20	−20
III.	76	54	108	112	−57	10	−6	−5	58	62	−8	−21
IV.	65	49	115	102	−100	35	25	−17	26	61	−37	−21
V.	14	35	106	72	54	98	−1	−47	78	48	36	−16
VI.	8	25	82	52	117	130	−44	−63	60	36	−3	−12
VII.	−16	−14	−46	−28	133	152	−116	−73	−90	−13	42	4
VIII.	−62	−45	−103	−93	78	57	−9	−27	−145	−57	−6	19
IX.	−43	−68	−106	−142	−118	−85	−46	−41				
(2) The 12-hour Component.												
I.	44	50	92	97	−55	−66	24	34	28	31	14	0
II.	49	54	96	105	−64	−58	24	30	41	38	6	0
III.	62	56	106	108	−7	−32	26	17	48	46	4	−1
IV.	62	55	102	106	−58	−15	60	8	39	48	−5	−1
V.	56	43	94	84	30	39	42	−20	49	44	16	−1
VI.	45	33	66	65	60	71	−18	−37	28	36	−26	0
VII.	−12	−19	−43	−37	60	95	−56	−49	−54	−13	−10	0
VIII.	−42	−52	−120	−100	53	1	−9	1	−32	−48	−2	1
IX.	−10	−47	−80	−92	−58	−68	0	35				

TABLE III. (α) (a) (continued).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(3) The 8-hour Component.												
I.	28	30	32	33	− 23	− 24	26	22	12	12	− 4	− 4
II.	36	37	49	41	− 24	− 26	30	23	22	17	2	− 6
III.	53	44	54	50	0	− 21	37	19	33	24	− 2	− 8
IV.	46	47	36	52	− 13	− 16	45	14	24	27	− 9	− 9
V.	56	43	46	48	12	9	24	− 8	31	28	− 2	− 9
VI.	42	35	20	39	21	24	− 17	− 22	12	25	− 30	− 8
VII.	− 16	− 20	− 28	− 23	25	34	− 21	− 30	− 32	− 10	− 6	3
VIII.	− 28	− 47	− 78	− 52	22	− 9	− 8	8	− 4	− 28	18	10
IX.	0	− 26	− 52	− 29	− 44	− 22	19	20				
(4) The 6-hour Component.												
I.	9	10	12	6	− 4	− 5	8	7	7	3	6	− 1
II.	14	14	16	8	− 4	− 6	10	9	9	4	0	− 2
III.	23	20	16	12	6	− 6	16	10	12	7	− 4	− 4
IV.	20	23	4	14	6	− 5	13	9	4	9	− 6	− 5
V.	28	24	6	14	4	0	5	− 1	12	11	− 7	− 6
VI.	18	21	− 4	12	3	5	− 7	− 8	0	10	− 12	− 5
VII.	− 12	− 8	− 11	− 5	6	11	− 2	− 18	− 13	− 4	0	2
VIII.	− 12	− 24	− 23	− 14	− 6	− 4	− 5	6	0	− 10	10	5
IX.	− 2	− 8	− 17	− 5	− 12	− 4	8	6				

TABLE III (α) (b).—Fourier Coefficients (Group Means) of the Solar Diurnal Magnetic Variation, at Sunspot Minimum, 1902.

(α) The Mean Equinoctial Component, $\frac{1}{2}$ (Spring + Autumn).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(1) The 24-hour Component.												
I.	58	62	78	87	−63	−45	28	32	2	45	−13	−5
II.	56	59	66	81	−81	−25	16	18	26	49	−4	−6
III.	65	51	70	70	−25	6	3	−4	38	51	0	−6
IV.	50	47	78	64	−82	22	30	−16	29	50	−18	−6
V.	17	33	42	45	−42	62	38	−45	65	45	8	−5
VI.	28	22	25	30	160	86	−25	−63	68	28	−20	−3
VII.	18	−8	−65	−11	152	101	−56	−73	−61	−10	−18	1
VIII.	−27	−49	−93	−67	−25	15	10	−11	−86	−50	−6	6
(2) The 12-hour Component.												
I.	42	48	64	69	−34	−47	34	33	20	19	4	−1
II.	46	52	64	74	−42	−41	26	28	29	24	0	−1
III.	60	53	72	77	1	−23	24	16	32	29	2	−1
IV.	72	52	70	75	−45	−11	48	7	30	31	−12	−1
V.	34	42	54	60	40	28	43	−19	40	31	8	−1
VI.	57	29	26	42	84	55	−22	−38	43	21	−26	−1
VII.	17	−11	−77	−16	66	74	−31	−51	38	−9	−25	0
VIII.	−10	−52	−111	−75	−4	−18	−4	13	0	−30	−8	−1

TABLE III (*a*) (*b*) (continued).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(3) The 8-hour Component.												
I.	31	27	28	23	−11	−17	26	20	12	9	−1	−4
II.	36	35	32	29	−18	−18	28	22	18	14	−2	−6
III.	55	42	33	35	5	−15	29	18	22	20	−6	−8
IV.	48	44	24	37	−4	−11	33	13	16	22	−8	−9
V.	24	44	24	37	24	6	36	−7	22	24	−8	−10
VI.	46	31	21	25	29	20	−26	−24	27	19	−26	−8
VII.	−3	−12	−38	−10	27	30	−22	−36	−22	−8	−10	3
VIII.	−16	−43	−55	−36	20	−13	−2	15	11	−21	9	9
(4) The 6-hour Component.												
I.	7	7	11	4	−2	−3	11	6	6	2	2	−2
II.	14	11	13	6	−2	−4	12	7	8	4	−1	−3
III.	24	16	11	8	6	−4	14	8	9	7	−6	−5
IV.	20	18	0	9	5	−4	10	7	3	8	−7	−6
V.	10	18	0	10	15	0	6	0	6	10	−4	−7
VI.	13	15	4	8	2	3	−14	−8	8	8	−12	−6
VII.	−10	−6	−10	−3	4	7	−4	−14	−12	−4	0	3
VIII.	−6	−17	−13	−9	−2	−3	0	5	2	−8	8	5

TABLE III. (β) (α).—Fourier Coefficients (Group Means) of the Solar Diurnal Magnetic Variation, at Sunspot Maximum, 1905.

(β) The Mean Solstitial Component, $\frac{1}{2}$ (Summer + Winter).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
	(1) The 24-hour Component.											
I.	54	56	107	133	−78	−68	46	29	11	49	−59	−16
II.	60	53	108	123	−98	−39	20	16	32	53	−17	−18
III.	66	46	104	107	−40	9	−8	−4	53	55	−8	−18
IV.	46	42	106	98	−64	34	26	−14	28	54	−27	−18
V.	5	29	93	69	52	94	−12	−40	54	43	22	−14
VI.	8	21	79	50	109	126	−49	−53	44	33	−4	−11
VII.	−18	−12	−54	−27	111	146	−100	−62	−66	−11	26	4
VIII.	−67	−38	−90	−89	66	54	−23	−23	−126	−51	−6	17
IX.	−55	−58	−100	−137	−80	−82	−4	35				
	(2) The 12-hour Component.											
I.	34	42	83	86	−51	−59	22	29	22	26	5	2
II.	38	46	80	93	−58	−51	8	25	38	33	6	2
III.	45	47	94	96	−16	−29	19	14	42	40	6	3
IV.	60	46	104	94	−48	−13	49	7	36	42	−3	3
V.	38	37	80	75	20	35	20	−17	36	38	17	3
VI.	24	28	64	58	45	63	−25	−31	28	31	−18	2
VII.	−26	−16	−46	−33	50	85	−54	−42	−46	−12	−8	−1
VIII.	−49	−44	−98	−89	48	1	−1	1	−24	−42	3	−3
IX.	−42	−40	−51	−81	−38	−60	34	30				

TABLE III. (β) (a) (continued).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
	(3) The 8-hour Component.											
I.	17	21	24	25	− 17	− 18	13	16	8	8	0	− 1
II.	22	27	29	31	− 16	− 20	20	17	15	12	2	− 2
III.	38	32	35	38	− 2	− 16	29	14	22	17	2	− 3
IV.	33	34	36	40	− 16	− 12	36	10	20	19	− 3	− 3
V.	33	31	39	36	2	7	18	− 6	20	20	2	− 3
VI.	32	26	23	30	12	19	− 12	− 16	13	18	− 18	− 3
VII.	− 16	− 15	− 25	− 17	25	26	− 21	− 22	− 20	− 7	− 5	1
VIII.	− 32	− 34	− 56	− 40	22	− 7	− 7	6	− 2	− 20	11	3
IX.	− 11	− 19	− 26	− 22	− 26	− 17	25	15				
	(4) The 6-hour Component.											
I.	2	4	6	4	0	− 3	1	3	4	1	0	− 1
II.	6	6	8	6	0	− 4	4	4	4	2	0	− 1
III.	10	9	8	9	4	− 4	10	4	5	4	1	− 2
IV.	10	10	2	10	0	− 4	10	4	4	4	− 2	− 2
V.	15	11	5	10	3	0	8	0	6	5	− 2	− 3
VI.	10	9	0	9	0	3	0	− 4	0	5	− 7	− 3
VII.	− 2	− 4	− 7	− 3	4	8	− 4	− 8	− 5	− 2	0	1
VIII.	− 5	− 11	− 19	− 10	7	− 3	− 6	3	0	− 5	6	3
IX.	0	− 3	− 16	− 3	− 11	− 3	9	3				

TABLE III. (β) (b).—Fourier Coefficients (Group Means) of the Solar Diurnal Magnetic Variation, at Sunspot Minimum, 1902.

(β) The Mean Solstitial Component, $\frac{1}{2}$ (Summer + Winter).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(1) The 24-hour Component.												
I.	53	56	74	79	− 49	− 41	24	29	16	40	− 16	− 9
II.	47	53	67	74	− 60	− 23	14	16	24	44	− 8	− 10
III.	57	46	64	64	− 16	6	− 3	− 4	38	46	− 7	− 10
IV.	44	42	68	59	− 50	21	28	− 14	26	45	− 17	− 10
V.	20	29	32	41	38	56	27	− 40	41	40	4	− 9
VI.	20	19	22	27	141	78	− 32	− 55	62	25	− 14	− 6
VII.	5	− 7	− 54	− 10	125	92	− 51	− 66	− 56	− 10	6	2
VIII.	− 26	− 43	− 91	− 61	− 18	14	10	− 10	− 70	− 45	− 8	10
(2) The 12-hour Component.												
I.	40	39	56	62	− 30	− 42	24	26	19	20	0	− 2
II.	38	42	54	67	− 40	− 37	17	23	24	25	− 1	− 3
III.	45	43	63	69	− 4	− 20	10	13	27	30	− 1	− 3
IV.	61	42	76	67	− 40	− 9	37	6	32	32	− 15	− 3
V.	29	33	47	54	30	25	30	− 16	26	32	2	− 3
VI.	36	24	20	38	70	49	− 23	− 31	36	22	− 14	− 2
VII.	10	− 9	− 62	− 14	56	66	− 30	− 41	− 36	− 9	− 14	1
VIII.	− 8	− 42	− 100	− 67	− 2	− 16	− 1	10	2	− 31	− 4	3

TABLE III. (β) (b) (continued).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(3) The 8-hour Component.												
I.	16	18	18	17	−11	−13	14	13	7	6	1	−3
II.	24	23	19	22	−12	−14	20	14	12	9	−2	−4
III.	37	27	22	26	1	−11	22	12	14	13	−4	−6
IV.	34	29	24	28	−12	−8	26	9	17	15	−10	−7
V.	16	29	29	28	18	5	14	−5	6	16	−12	−8
VI.	32	20	11	19	19	15	−19	−15	19	12	−12	−6
VII.	4	−8	−26	−8	23	23	−20	−23	−17	−5	−4	3
VIII.	−10	−28	−46	−27	0	−10	−2	10	7	−14	7	7
(4) The 6-hour Component.												
I.	2	3	4	2	−1	−1	1	3	4	1	0	−1
II.	4	5	4	3	−1	−2	6	3	4	2	0	−1
III.	10	7	3	4	2	−2	7	3	2	3	−1	−2
IV.	10	8	2	4	1	−2	10	3	4	3	−6	−3
V.	7	8	6	5	6	0	2	0	1	4	−1	−3
VI.	10	7	−3	4	0	2	−2	−3	2	3	−7	−3
VII.	0	−3	−6	−2	5	3	−1	−6	−2	−1	−4	−1
VIII.	2	−7	−10	−5	4	−2	−2	3	1	−3	4	−3

TABLE III. (γ) (α).—Fourier Coefficients (Group Means) of the Solar Diurnal Magnetic Variation, at Sunspot Maximum, 1905.

(γ) The Solstitial Inequality, $\frac{1}{2}$ (Summer − Winter).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(1) The 24-hour Component.												
I.	7	4	75	84	−48	−32	32	19	14	30	1	7
II.	3	7	73	76	−42	−12	36	26	24	29	8	8
III.	10	12	66	64	−18	14	34	33	29	23	8	6
IV.	16	15	36	57	30	25	29	35	4	19	−1	5
V.	13	21	47	42	−1	36	20	32	4	3	4	1
VI.	22	24	34	34	21	33	12	27	−1	−7	−10	−1
VII.	34	27	46	28	−6	−22	17	−15	9	−16	10	−4
VIII.	31	17	70	52	−31	−31	18	−35	48	13	12	4
IX.	21	2	86	87	66	40	−27	−16				
(2) The 12-hour Component.												
I.	36	36	40	32	−24	−19	26	26	12	13	−4	1
II.	42	39	36	32	−20	−15	9	24	24	15	2	0
III.	43	43	40	32	8	−6	27	20	26	15	6	−1
IV.	36	44	10	31	32	−1	31	17	12	14	3	−2
V.	36	43	18	24	10	8	14	8	6	7	−9	−5
VI.	46	43	4	20	23	11	9	4	2	−1	−14	−8
VII.	38	42	46	16	−2	−7	12	−2	12	−5	0	−10
VIII.	54	44	38	19	−26	−3	1	−14	−14	12	−9	−3
IX.	43	34	32	30	12	20	−18	−26				

TABLE III. (γ) (a) (continued).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(3) The 8-hour Component.												
I.	3	6	9	7	0	7	6	6	8	2	4	1
II.	8	9	10	7	1	6	1	8	7	3	3	2
III.	12	14	11	6	14	3	17	12	10	4	2	1
IV.	21	16	− 6	4	20	1	20	13	2	5	1	0
V.	23	24	− 15	− 2	6	− 5	6	13	0	5	− 10	− 5
VI.	26	28	− 13	− 7	9	− 6	8	12	− 1	5	− 10	− 9
VII.	18	31	27	− 11	− 5	5	9	− 7	14	4	− 7	− 13
VIII.	38	19	8	2	− 12	1	11	− 13	− 10	5	− 9	− 2
IX.	7	5	4	6	0	− 7	− 14	− 5				
(4) The 6-hour Component.												
I.	− 2	− 2	− 4	− 4	4	3	− 4	− 2	2	0	2	1
II.	− 7	− 3	− 6	− 6	4	4	− 4	− 2	0	0	0	1
III.	− 8	− 4	− 8	− 9	6	6	0	− 1	− 3	− 1	1	1
IV.	− 6	− 4	− 14	− 11	12	6	6	− 1	− 4	− 1	2	1
V.	− 3	− 4	− 23	− 16	3	5	− 4	1	− 6	− 3	− 8	− 1
VI.	− 6	− 3	− 14	− 17	0	4	0	2	− 2	− 4	3	− 3
VII.	4	− 2	− 7	− 19	0	− 1	4	− 1	3	− 5	− 4	− 5
VIII.	11	− 4	− 11	− 12	− 3	− 6	6	0	− 2	− 1	1	0
IX.	0	− 2	0	− 3	− 2	− 3	− 2	1				

Table III. (γ) (b).—Fourier Coefficients (Group Means) of the Solar Diurnal Magnetic Variation at Sunspot Minimum, 1902.

(γ) The Solstitial Inequality, $\frac{1}{2}$ (Summer—Winter).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(1) The 24-hour Component.												
I.	15	8	64	72	−44	−32	20	16	16	21	2	7
II.	6	10	62	66	−36	−16	21	18	24	21	1	7
III.	9	12	60	56	−15	5	17	21	22	19	11	5
IV.	20	14	39	52	7	14	27	22	10	17	0	4
V.	8	17	24	40	17	25	34	20	10	12	−8	2
VI.	25	19	50	33	9	23	−2	14	12	3	4	−3
VII.	13	20	53	28	−2	−9	−9	−6	0	−1	9	−5
VIII.	12	13	73	54	13	−10	5	−1	13	17	14	4
(2) The 12-hour Component.												
I.	31	29	31	33	−14	−18	20	19	12	10	−2	1
II.	32	31	26	34	−16	−12	13	17	20	12	0	1
III.	33	33	31	31	10	0	18	12	15	13	5	0
IV.	33	33	9	29	20	7	25	9	16	13	−1	−1
V.	17	30	5	18	32	17	20	2	8	12	−4	−4
VI.	42	28	18	11	10	18	3	−2	15	7	−12	−8
VII.	28	27	21	3	0	−8	0	1	2	4	−4	−10
VIII.	30	32	50	29	8	−4	5	−10	−14	13	−4	−1

TABLE III. (γ) (b) (continued).

Group No.	West.				North.				Radial.			
	a.		b.		a.		b.		a.		b.	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(3) The 8-hour Component.												
I.	6	6	8	4	− 1	2	8	6	5	2	3	1
II.	8	9	5	4	2	2	4	8	6	3	1	1
III.	13	13	8	4	13	0	12	10	6	5	4	1
IV.	18	16	− 6	4	16	− 1	20	11	7	6	0	0
V.	14	22	− 7	1	20	− 3	2	11	8	6	0	− 2
VI.	30	26	− 5	− 1	3	− 4	5	9	6	7	− 14	− 6
VII.	25	29	2	− 3	1	2	3	− 4	3	7	− 4	− 9
VIII.	26	18	16	4	2	1	2	− 11	− 5	6	− 7	0
(4) The 6-hour Component.												
I.	− 2	− 1	0	− 2	0	2	− 1	− 2	2	0	2	1
II.	− 4	− 2	− 2	− 3	3	2	− 4	− 2	0	0	0	2
III.	− 4	− 1	− 3	− 5	6	4	− 1	− 2	− 2	0	2	2
IV.	− 2	− 1	− 12	− 6	11	3	6	− 1	0	0	2	1
V.	3	1	− 12	− 9	2	0	− 6	1	− 3	− 2	− 1	0
VI.	2	4	− 9	− 10	− 2	− 2	6	2	− 4	− 3	− 3	− 4
VII.	5	5	− 6	− 11	2	1	3	− 1	2	− 3	− 7	− 6
VIII.	12	− 1	− 6	− 6	0	− 3	0	1	− 1	− 1	2	1

TABLE III. (δ).—Fourier Coefficients of the Solar Diurnal Magnetic Variation, Combined Group Means for 1902 and 1905.

(δ) The Unsymmetrical Seasonal Component, ½ (Spring—Autumn).

Group No.	West.				North.				Radial.			
	a.		b.		a.		b.		a.		b.	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(1) The 24-hour Component.												
I.	− 6	− 7	− 8	− 11	5	10	− 16	− 11	4	2	− 2	− 3
II.	− 9	− 8	− 11	− 11	12	11	− 19	− 13	2	2	− 1	− 3
III.	− 13	− 10	− 12	− 12	24	11	− 21	− 15	4	2	− 1	− 2
IV.	− 4	− 11	− 6	− 12	10	11	− 21	− 16	3	2	− 3	− 1
V.	− 11	− 13	− 13	− 13	12	9	− 15	− 14	2	1	6	0
VI.	− 18	− 14	− 15	− 13	17	6	− 6	− 11	− 3	0	4	2
VII.	− 27	− 15	− 18	− 14	5	− 4	17	6	− 10	− 1	− 8	3
VIII.	− 12	− 11	− 20	− 12	5	− 11	− 5	15	− 11	1	− 8	− 1
IX.	− 1	− 6	− 18	− 11	− 13	− 11	1	10				
(2) The 12-hour Component.												
I.	− 23	− 21	4	1	− 2	0	− 16	− 16	− 1	0	5	− 1
II.	− 23	− 23	2	1	− 2	0	− 18	− 16	− 2	0	2	0
III.	− 25	− 27	0	0	3	1	− 24	− 15	1	− 1	1	1
IV.	− 26	− 28	2	0	− 13	1	− 22	− 14	− 3	− 1	2	3
V.	− 31	− 30	0	− 1	− 5	2	− 8	− 10	− 3	− 3	9	7
VI.	− 37	− 31	3	− 1	4	2	− 6	− 6	− 3	− 5	14	10
VII.	− 32	− 31	− 7	− 2	5	− 1	6	4	− 6	− 6	0	12
VIII.	− 18	− 28	− 11	0	4	− 1	− 3	13	2	− 1	1	3
IX.	− 16	− 19	− 10	2	− 1	0	1	16				

TABLE III. (δ) (continued).

Group No.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(3) The 8-hour Component.												
I.	−9	−7	7	4	−4	−3	−5	−6	2	1	2	1
II.	−8	−9	7	6	−9	−4	−6	−8	−1	1	2	1
III.	−10	−13	8	7	−6	−5	−14	−10	2	1	0	3
IV.	−16	−16	8	8	−14	−5	−12	−10	−2	1	2	4
V.	−22	−22	12	11	−5	−4	−2	−10	1	1	8	6
VI.	−29	−25	18	11	−2	−3	−2	−8	2	0	10	8
VII.	−16	−27	0	10	1	2	0	5	−4	−1	4	9
VIII.	−16	−16	1	13	−8	5	0	10	−1	1	6	4
IX.	−10	−6	2	8	−6	3	−1	5				
(4) The 6-hour Component.												
I.	−2	−1	3	2	−1	−1	0	−1	1	0	0	0
II.	−1	−2	5	3	−3	−3	−2	−1	0	1	1	0
III.	−1	−3	6	5	−3	−4	−2	−2	2	1	0	1
IV.	−3	−3	7	6	−8	−5	−2	−2	0	1	1	1
V.	−5	−4	12	11	−4	−5	3	−1	3	2	4	2
VI.	−4	−5	15	14	−3	−5	0	−1	2	1	1	3
VII.	−4	−5	8	15	−2	2	−4	0	−1	1	4	4
VIII.	−5	−4	−3	7	−1	5	0	2	1	1	1	1
IX.	−6	−1	1	2	2	1	0	1				

TABLE IV. (1) (*a*).*—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to the Epoch of New Moon.

(1) Zi-Ka-Wei.

(*a*) West Declination (in Force Units).

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer	1	167	−34	283	−125	127	−113	14	−26
	2	153	−43	287	2	117	−30	2	−3
	3	125	37	235	35	112	3	5	−17
	4	93	−18	179	−9	114	−3	35	−21
	5	65	−30	209	−18	97	−44	4	−12
	6	181	55	270	−17	116	−32	11	−4
	7	86	−5	209	6	108	0	19	−3
	8	54	5	207	22	89	−13	−21	3
	Mean. .	116	−4	235	−13	110	−29	9	−10
Equinox	1	56	15	200	−17	101	−33	29	−37
	2	40	16	197	−7	103	−6	25	−1
	3	48	15	143	−34	61	−38	13	−16
	4	22	29	113	−4	98	−11	24	−28
	5	157	37	165	−46	78	−5	25	−9
	6	50	72	131	−2	84	−20	38	−16
	7	36	−9	211	−49	110	−63	23	−18
	8	81	−23	119	−16	57	5	26	0
	Mean. .	61	19	160	−22	86	−21	25	−16
Winter	1	21	−41	−13	−82	−22	−41	23	−12
	2	−34	49	−4	−14	−46	−15	−15	−2
	3	−8	31	−20	−50	−19	−21	8	−28
	4	−21	−16	−38	−39	−40	−21	−18	−2
	5	−49	11	−30	−55	−23	−36	8	−17
	6	44	−6	22	−103	−16	−55	−4	−22
	7	−16	−41	20	−54	−19	−27	10	10
	8	56	−24	65	−96	26	−44	−17	−36
	Mean. .	−1	−5	0	−62	−20	−32	−1	−14

* For an explanation of Tables IV., V., and VI., *cf.* § 12. The unit of force in these three tables is 10^{-7} C.G.S.

TABLE IV. (1) (*b*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to the Epoch of New Moon.

(1) Zi-Ka-Wei.

(*b*) Horizontal Force.

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer	1	40	33	112	−24	43	−6	−8	−20
	2	−13	67	14	46	50	11	4	8
	3	−13	−20	38	−55	36	27	−13	−2
	4	−83	28	−3	12	−17	22	−25	25
	5	41	54	−35	−31	−6	−18	−1	1
	6	−48	68	6	−34	30	36	−3	20
	7	6	53	30	15	2	9	1	21
	8	−15	18	8	8	35	24	7	3
	Mean. .	−11	38	21	−8	22	13	−5	7
Equinox	1	−33	22	13	5	10	44	−13	10
	2	68	27	38	61	38	31	−4	34
	3	−37	125	−24	−9	18	23	12	18
	4	−42	4	−19	−35	50	1	23	32
	5	−137	160	1	76	39	83	12	25
	6	151	5	40	54	0	38	5	23
	7	−60	49	45	85	62	6	−15	4
	8	−116	−83	2	−20	−24	3	−19	9
	Mean. .	−26	39	12	27	24	29	0	19
Winter	1	55	−38	108	−77	25	−20	−2	25
	2	−74	−89	15	−154	3	−42	2	−13
	3	45	−123	110	−175	37	−28	12	9
	4	94	−182	119	−139	40	−52	23	−9
	5	91	−64	82	−61	24	−3	−7	17
	6	95	−9	85	−100	68	−19	44	4
	7	158	−14	53	−56	26	6	−1	−18
	8	105	−13	74	−101	−1	−32	3	16
	Mean. .	71	−66	81	−108	28	−24	9	4

TABLE IV. (1) (*c*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to the Epoch of New Moon.

(1) Zi-Ka-Wei.

(*c*) Vertical Force (Upwards).

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer . . .	1	41	− 39	87	− 15	− 34	− 45	− 18	− 9
	2	15	− 92	130	− 29	12	− 62	13	− 24
	3	47	− 37	116	− 1	14	− 29	8	1
	4	65	− 58	112	36	2	− 35	15	− 22
	5	− 12	− 27	80	11	− 3	− 17	− 4	6
	6	41	− 41	125	− 38	14	− 28	30	− 16
	7	− 5	20	118	− 4	14	− 48	− 8	− 17
	8	11	− 56	119	6	− 37	− 26	− 21	− 3
	Mean. .	26	− 41	111	− 4	− 2	− 36	2	− 11
Equinox . . .	1	14	23	98	28	− 5	− 4	− 6	3
	2	58	30	94	51	− 2	− 10	− 7	− 6
	3	28	− 12	119	31	− 1	− 20	− 12	− 17
	4	− 65	108	110	− 34	34	1	− 29	7
	5	32	− 160	190	135	− 1	− 70	38	40
	6	65	42	141	− 6	15	− 11	19	− 25
	7	32	− 21	99	− 11	− 19	− 37	4	− 14
	8	19	− 35	127	9	6	− 18	− 6	− 1
	Mean. .	23	− 3	122	25	4	− 21	0	− 2
Winter . . .	1	− 59	− 33	42	8	− 26	− 6	− 8	− 17
	2	− 60	− 68	43	− 23	7	− 34	23	− 10
	3	52	− 33	85	76	− 24	14	− 10	− 14
	4	− 98	65	65	− 47	43	2	− 27	8
	5	20	− 56	133	32	12	− 10	16	− 13
	6	51	− 92	50	2	− 10	− 8	− 5	1
	7	− 39	− 52	110	36	− 3	− 21	− 4	− 6
	8	− 89	− 8	114	64	28	4	20	− 34
	Mean. .	− 28	− 34	80	18	4	− 7	0	− 11

TABLE IV. (2) (*a*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to the Epoch of New Moon.

(2) Manila.

(*a*) West Declination (in Force Units).

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer . . .	1	92	−45	114	−146	42	−83	19	−10
	2	104	−71	145	−47	64	−28	−11	8
	3	127	36	162	9	100	0	−3	2
	4	71	−20	127	−17	93	−6	34	−12
	5	5	11	102	−30	59	−58	−3	−20
	6	156	7	181	−65	75	−61	−3	−23
	7	32	17	141	−3	80	−20	16	17
	8	36	19	133	−3	79	−23	6	−21
	Mean. .	78	−6	138	−38	74	−35	7	−7
Equinox . . .	1	82	−17	128	−70	67	−31	15	−9
	2	13	40	128	−21	115	−16	36	4
	3	44	27	116	−32	68	−43	18	−22
	4	56	29	86	−28	66	−38	23	−35
	5	77	37	88	−67	44	−19	25	−1
	6	37	39	57	−34	47	−14	22	−15
	7	17	−5	146	−53	85	−67	20	−11
	8	94	14	84	−54	36	−13	12	−17
	Mean. .	52	20	104	−45	66	−30	21	−13
Winter . . .	1	17	−74	−59	−90	−37	−31	20	−10
	2	−61	34	−83	−43	−70	−11	−30	−1
	3	−46	0	−108	−92	−54	−35	1	−17
	4	−38	−53	−120	−97	−75	−48	−26	−13
	5	−43	−76	−92	−118	−73	−52	−17	−5
	6	−60	−30	−102	−125	−53	−63	−6	−27
	7	4	−24	−54	−74	−46	−28	−4	15
	8	24	−39	−20	−134	−19	−60	10	−20
	Mean. .	−25	−33	−80	−97	−53	−41	−6	−10

TABLE IV. (2) (*b*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to the Epoch of New Moon.

(2) Manila.

(*b*) Horizontal Force.

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer	1	− 57	− 147	− 64	− 64	− 76	− 4	− 2	32
	2	− 153	− 38	− 32	− 30	− 10	− 36	11	0
	3	− 12	− 24	− 39	− 122	− 7	− 2	− 14	1
	4	6	− 15	− 4	− 48	9	− 27	− 30	6
	5	− 118	− 77	− 123	− 23	− 59	− 12	9	22
	6	− 146	− 58	24	− 70	− 20	− 8	0	− 10
	7	58	69	− 42	− 69	− 45	− 13	0	− 4
	8	79	− 54	− 6	− 51	3	− 23	− 4	− 1
	Mean. .	− 43	− 43	− 36	− 60	− 26	− 16	− 4	6
Equinox	1	− 14	− 146	− 53	− 66	− 34	0	− 34	20
	2	− 80	− 61	55	− 56	6	− 41	− 12	11
	3	− 23	98	− 77	− 57	− 8	− 25	2	25
	4	− 18	− 78	− 50	− 80	26	2	4	19
	5	− 128	33	− 61	− 16	11	6	− 10	− 4
	6	96	19	27	− 34	− 12	− 16	− 22	− 10
	7	24	68	− 47	− 63	0	− 61	− 33	− 19
	8	28	− 138	6	− 25	− 13	− 27	− 28	− 9
	Mean. .	− 14	− 26	− 25	− 50	− 3	− 20	− 17	4
Winter	1	− 44	− 134	− 46	− 14	− 86	− 21	− 46	11
	2	− 183	− 117	− 54	− 88	− 55	− 4	− 11	7
	3	17	− 40	− 19	− 177	− 32	− 30	− 7	− 15
	4	131	− 252	− 30	− 162	− 24	− 47	− 4	− 19
	5	− 17	− 174	− 4	− 135	− 5	− 52	− 29	− 14
	6	− 161	− 138	− 47	− 168	− 64	− 32	− 10	13
	7	73	− 1	− 74	− 163	− 26	− 46	− 32	− 16
	8	53	− 126	− 137	− 164	− 92	− 38	− 25	− 2
	Mean. .	− 16	− 123	− 51	− 134	− 48	− 34	− 20	− 4

TABLE IV. (2) (*c*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to the Epoch of New Moon.

(2) Manila.

(*c*) Vertical Force (Upwards).

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer	1	-4	-58	-38	-80	-26	-33	11	-2
	2	38	-64	7	-87	-3	-41	0	10
	3	63	-58	42	-79	10	-36	4	4
	4	19	-2	18	-94	37	-31	-13	-6
	5	9	-20	-14	-47	-27	-38	-13	-3
	6	-24	-94	3	-93	-5	-29	-3	-1
	7	15	-15	15	-77	9	-43	2	-5
	8	45	-68	28	-95	1	-45	-17	-4
	Mean	20	-47	8	-81	0	-37	-4	-1
Equinox	1	38	-42	11	-123	-9	-70	-12	-17
	2	19	-13	19	-112	35	-74	14	-23
	3	50	-42	16	-122	7	-70	3	-11
	4	34	-69	0	-74	-19	-62	-8	-13
	5	-2	-64	-15	-122	5	-56	6	-23
	6	16	-75	10	-100	10	-52	11	-13
	7	-20	-43	5	-69	-2	-55	0	-19
	8	19	-37	16	-79	4	-32	-2	-2
	Mean	19	-48	8	-100	4	-59	2	-15
Winter	1	-80	-41	-80	-46	-49	-14	-13	-6
	2	-55	-16	-93	-32	-49	-13	-10	-10
	3	-44	-48	-62	-75	-41	-30	-6	-7
	4	66	12	-63	-45	-32	-7	-19	1
	5	-58	-43	-47	-85	-24	-44	-5	-15
	6	-25	-58	-74	-109	-40	-61	-14	-16
	7	8	-22	-29	-97	-30	-42	-14	-7
	8	-21	-124	-75	-112	-50	-47	-9	-13
	Mean	-26	-42	-65	-73	-39	-32	-11	-9

TABLE IV. (3) (*a*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to the Epoch of New Moon.

(3) Batavia.

(*a*) West Declination (in Force Units).

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer . . .	1	− 8	4	− 10	− 24	− 4	− 2	8	0
	2	10	− 3	35	− 56	2	− 50	− 9	− 21
	3	57	67	37	20	− 13	7	− 3	5
	4	− 6	2	6	− 32	19	3	3	− 3
	5	11	− 4	45	0	2	17	− 6	− 3
	6	38	55	34	− 10	− 2	− 3	− 24	− 15
	7	− 7	− 33	6	− 23	− 12	4	− 3	11
	8	− 17	55	29	2	14	7	− 9	− 15
	Mean . .	9	18	23	− 15	1	− 2	− 5	− 5
Equinox . . .	1	4	− 39	− 42	− 34	− 28	− 3	− 6	26
	2	− 55	− 50	− 153	− 2	− 37	− 2	− 25	24
	3	24	17	− 20	9	− 38	− 9	− 28	− 1
	4	− 28	51	− 102	38	− 18	6	− 42	3
	5	− 26	− 4	− 48	32	− 20	6	− 31	25
	6	− 41	− 27	− 95	− 13	− 55	− 15	− 37	− 21
	7	20	− 50	− 45	− 50	− 14	− 30	− 17	− 14
	8	− 7	− 12	− 21	− 44	− 10	− 6	− 31	− 18
	Mean . .	− 14	− 14	− 66	− 8	− 28	− 7	− 27	3
Winter . . .	1	− 109	− 70	− 262	59	− 113	112	− 21	56
	2	− 87	− 28	− 207	78	− 32	123	− 8	37
	3	− 119	− 52	− 246	− 63	− 126	55	− 38	42
	4	− 93	− 75	− 225	14	− 82	37	− 31	44
	5	− 60	− 46	− 256	15	− 118	30	− 4	33
	6	− 88	− 84	− 312	57	− 172	61	− 70	46
	7	− 56	− 69	− 301	18	− 157	55	− 80	32
	8	− 126	− 4	− 262	30	− 116	61	− 27	15
	Mean . .	− 92	− 54	− 259	26	− 114	67	− 35	38

TABLE IV. (3) (*b*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to the Epoch of New Moon.

(3) Batavia.

(*b*) Horizontal Force.

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer . . .	1	− 19	− 32	− 66	− 70	28	9	− 6	22
	2	− 83	− 114	− 54	− 78	− 22	− 42	20	10
	3	47	− 70	− 20	− 108	9	− 40	12	11
	4	− 50	− 54	7	− 37	− 1	− 27	− 7	− 5
	5	10	92	− 71	− 62	− 3	2	14	18
	6	38	− 41	− 64	− 88	1	16	3	12
	7	40	− 28	− 35	− 87	14	3	21	− 9
	8	− 8	− 68	− 42	− 64	9	− 18	− 21	11
	Mean. .	− 3	− 39	− 43	− 74	4	− 12	4	9
Equinox. . .	1	47	− 89	− 57	− 49	− 22	− 24	− 29	6
	2	30	8	− 93	− 64	− 10	− 2	− 23	3
	3	− 38	36	− 82	32	− 29	12	− 21	− 20
	4	− 103	22	− 42	− 71	20	− 29	21	0
	5	− 28	63	− 47	− 1	8	0	− 1	− 8
	6	96	− 108	− 17	− 87	− 9	− 17	− 22	− 9
	7	− 31	− 64	1	− 66	17	− 37	11	1
	8	− 102	− 137	− 154	− 53	− 42	− 7	− 11	23
	Mean. .	− 16	− 34	− 61	− 45	− 8	− 13	− 9	0
Winter . . .	1	− 59	− 57	− 64	− 128	− 32	− 27	− 12	− 8
	2	− 48	− 26	− 94	− 77	− 24	− 28	− 14	14
	3	59	− 2	− 87	− 118	− 31	− 39	5	− 6
	4	30	− 18	− 54	− 123	− 23	− 43	− 20	− 35
	5	− 7	30	− 90	− 93	− 24	− 4	− 17	− 20
	6	136	− 77	− 118	− 109	− 26	− 3	14	− 27
	7	78	− 23	− 88	− 123	− 22	− 8	− 40	4
	8	− 26	− 25	− 95	− 139	− 31	− 37	− 36	− 36
	Mean. .	20	− 22	− 86	− 112	− 27	− 38	− 15	− 14

TABLE IV. (3) (*c*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to the Epoch of New Moon.

(3) Batavia.

(*c*) Vertical Force (Upwards).

Season.	Lunar phase.	a'_1.	b'_1.	a_2.	b_2.	a'_3.	b'_3.	a'_4.	b'_4.
Summer	1	− 6	− 36	− 12	25	− 5	0	− 12	5
	2	− 21	− 4	18	49	15	11	15	5
	3	− 3	− 22	8	35	− 1	− 13	− 3	− 7
	4	− 8	− 16	33	73	− 13	− 5	8	− 5
	5	− 9	9	− 21	34	− 4	7	7	6
	6	− 13	− 47	− 3	26	− 8	− 1	5	− 2
	7	18	− 12	14	43	7	− 7	3	− 1
	8	− 21	− 20	− 3	23	8	− 8	7	1
	Mean	− 8	− 19	4	38	0	− 2	4	0
Equinox	1	44	− 24	34	18	6	− 6	− 12	− 5
	2	42	− 11	19	− 10	25	− 11	− 2	− 7
	3	− 8	8	− 4	48	6	− 7	2	− 11
	4	8	6	10	19	4	8	3	6
	5	38	12	3	33	− 2	− 20	2	− 8
	6	16	− 41	27	23	20	− 4	16	− 9
	7	7	− 7	32	22	17	− 8	1	0
	8	− 16	− 53	14	58	0	7	21	− 6
	Mean	16	− 14	16	26	10	− 5	4	− 5
Winter	1	23	− 47	8	− 69	− 20	− 55	− 25	− 15
	2	− 16	− 78	− 31	− 40	− 34	− 45	− 17	− 1
	3	64	− 21	33	− 19	− 18	− 31	− 7	− 22
	4	− 17	− 31	− 10	− 22	− 11	− 27	− 12	− 17
	5	12	22	22	− 19	18	− 8	− 8	− 10
	6	− 2	− 25	− 18	− 17	7	− 37	8	− 37
	7	49	− 42	18	− 21	7	− 38	− 5	− 9
	8	11	19	13	− 28	− 3	− 52	− 14	− 26
	Mean	16	− 25	5	− 29	− 7	− 37	− 10	− 17

TABLE V.—Fourier Coefficients (in the Formula $\Sigma C_n \sin(nt+\theta_n)$) of the Lunar Diurnal Magnetic Variation, Reduced to the Epoch of New Moon, at Pavlovsk, Pola, Zi-Ka-Wei, Manila, and Batavia.

Observatory.	Force component.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
			°		°		°		°
Summer.									
Pavlovsk	West	131	114	128	88	28	83	2	—
	North	61	20	105	4	62	355	26	291
	Radial	36	145	13	203	11	282	1	—
Pola	West	113	124	161	78	80	86	4	269
	North	111	12	133	25	86	41	20	46
	Radial	15	126	62	129	30	131	6	126
Zi-Ka-Wei	West	117	81	236	83	117	94	14	130
	North	42	328	14	112	23	40	10	316
	Radial	50	138	111	82	37	173	12	159
Manila	West	80	85	144	97	84	107	11	128
	North	61	216	69	201	30	228	7	318
	Radial	52	149	82	166	38	172	4	248
Batavia	West	20	18	27	117	2	99	8	219
	North	40	177	86	203	13	153	11	19
	Radial	10	195	38	359	2	174	4	84
Equinox.									
Pavlovsk	West	79	116	82	84	13	96	15	115
	North	42	345	48	359	63	1	27	338
	Radial	13	46	17	15	2	—	5	229
Pola	West	71	124	60	60	48	100	26	107
	North	63	3	80	29	71	37	21	53
	Radial	14	97	24	164	14	166	14	171
Zi-Ka-Wei	West	65	61	162	87	91	69	32	110
	North	48	313	28	0	37	25	22	347
	Radial	24	88	125	68	22	160	2	173
Manila	West	58	60	114	105	74	106	28	113
	North	29	200	56	197	21	177	18	275
	Radial	53	150	100	167	60	168	17	166
Batavia	West	20	216	66	256	29	249	30	269
	North	38	198	76	226	16	205	10	266
	Radial	22	123	31	25	11	111	7	134

TABLE V.—(continued).

Observatory.	Force component.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
			°		°		°		°
Winter.									
Pavlovsk	West	28	214	36	310	21	304	15	131
	North	17	172	20	246	20	245	2	—
	Radial	28	284	6	157	2	—	4	47
Pola	West	46	218	45	320	15	320	3	349
	North	50	144	30	126	11	159	2	—
	Radial	15	123	35	118	7	89	1	—
Zi-Ka-Wei	West	8	197	66	167	39	199	15	170
	North	99	123	133	132	37	118	11	53
	Radial	45	208	82	66	8	144	12	167
Manila	West	41	211	123	212	68	224	13	204
	North	125	179	145	193	61	227	23	250
	Radial	51	203	98	157	41	222	16	223
Batavia	West	109	233	260	268	135	293	67	310
	North	31	130	141	210	48	208	23	219
	Radial	30	142	30	164	38	183	33	202

TABLE VI. (*a*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to New Moon, at the Solstices.

No. of observatory.	West.				North.				Radial.			
	a.		*b*.		*a*.		*b*.		*a*.		*b*.	
	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.	Summer.	Winter.
(1) The 24-hour Term.												
1	120	− 16	− 53	− 23	20	2	57	− 17	20	− 27	− 30	7
2	94	− 29	− 63	− 36	24	30	108	− 40	12	13	− 9	− 8
3	116	− 3	19	− 8	− 22	84	35	− 53	33	− 21	− 37	− 40
4	80	− 21	6	− 35	− 36	1	− 50	− 125	27	− 20	− 45	− 47
5	6	− 86	19	− 66	2	24	− 40	− 20	3	− 19	10	24
(2) The 12-hour Term.												
1	128	− 28	3	23	8	− 18	105	− 8	− 5	2	− 12	− 6
2	157	− 29	34	35	57	24	120	− 18	48	31	− 39	− 17
3	234	15	30	− 64	13	98	− 5	− 89	110	75	16	33
4	143	− 64	− 17	− 105	− 25	− 33	− 64	− 142	19	− 54	− 80	− 81
5	24	− 260	− 12	− 8	− 33	− 71	− 79	− 122	1	− 9	− 38	29
(3) The 8-hour Term.												
1	28	− 17	3	12	− 6	− 18	62	− 8	− 11	− 2	2	0
2	80	− 10	5	11	57	4	65	− 10	23	7	− 20	0
3	116	− 13	− 8	− 37	15	33	18	− 17	4	5	− 37	− 7
4	80	− 47	− 24	− 49	− 22	− 44	− 20	− 42	5	− 28	− 38	− 30
5	2	− 124	0	52	6	− 23	− 12	− 42	0	2	2	38
(4) The 6-hour Term.												
1	2	11	1	− 10	− 24	0	9	2	1	3	1	− 3
2	− 4	0	0	3	14	1	14	− 2	4	− 1	− 3	0
3	11	3	− 9	− 15	− 7	9	7	7	4	3	− 11	− 12
4	9	− 4	− 7	− 11	− 5	− 22	6	− 8	− 4	− 11	− 2	− 12
5	− 5	− 52	− 6	43	4	− 14	10	− 18	− 4	13	0	31

TABLE VI. (*b*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to New Moon, at the Equinoxes.

No. of observatory.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(1) The 24-hour Term.												
1	71	88	−35	−1	−11	1	40	45	10	23	9	−12
2	59	72	−40	−1	3	0	63	0	14	26	−1	−14
3	57	53	32	−1	−35	−1	33	−42	23	23	1	−12
4	50	26	29	0	−10	−1	−28	−80	26	13	−46	−7
5	−12	−11	−16	0	−12	−1	−36	−90	−18	−5	12	3
(2) The 12-hour Term.												
1	81	118	9	9	−1	−7	48	84	4	27	16	−5
2	52	135	30	10	39	−4	70	47	6	43	−23	−8
3	162	120	8	9	0	2	28	−22	115	47	48	−9
4	110	66	−29	5	−16	8	−53	−106	22	29	−98	−5
5	−64	−29	−16	−2	−55	10	−53	−130	−13	−13	−27	2
(3) The 8-hour Term.												
1	13	41	−1	2	1	−2	63	31	1	4	1	−10
2	47	66	−9	3	43	−2	56	31	3	8	−13	−23
3	85	71	33	4	15	0	33	−3	7	11	−20	−30
4	71	44	−20	2	1	2	−21	−42	12	7	−59	−21
5	−27	−20	−10	−1	−7	3	−14	−58	−10	−3	4	10
(4) The 6-hour Term.												
1	14	13	−6	−4	−10	3	25	11	−4	0	−3	−3
2	25	30	−8	−9	17	5	13	16	2	2	−14	−8
3	30	39	−11	−11	−5	2	21	6	0	2	−2	−13
4	26	28	−11	−8	−18	−5	2	−19	4	2	−16	−11
5	−30	−13	−1	4	−10	−8	−1	−28	−5	−1	5	5

TABLE VI. (*c*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to New Moon. Mean Solstitial Component, $\frac{1}{2}$ (Summer + Winter).

No. of observatory.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
(1) The 24-hour Component.												
1	52	74	−38	−25	11	13	20	42	−4	7	−12	−30
2	32	61	−50	−21	27	0	34	0	12	7	−8	−35
3	56	45	6	−15	31	−14	−9	−39	6	7	−38	−31
4	30	22	−14	−7	−18	−26	−88	−75	4	4	−46	−17
5	−40	−9	−24	3	13	−29	−30	−84	−8	−2	17	7
(2) The 12-hour Component.												
1	50	96	13	−5	−5	4	48	69	−2	20	−9	−15
2	64	110	34	−6	40	2	51	39	40	32	−28	−24
3	110	98	−17	−5	56	−1	−47	−18	92	34	24	−26
4	40	54	−61	−3	−29	−5	−103	−87	−18	21	−80	−16
5	−118	−24	−10	1	−52	−6	−100	−107	−4	−10	−4	7
(3) The 8-hour Component.												
1	6	29	8	−11	−12	9	27	22	−6	0	1	−9
2	35	46	8	−18	30	9	28	22	15	0	−10	−21
3	52	50	−22	−20	24	−1	0	0	4	0	−22	−27
4	16	31	−36	−12	−33	−12	−31	−30	−12	0	−34	−19
5	−61	−14	26	6	−8	−16	−27	−41	1	0	20	9
(4) The 6-hour Component.												
1	6	4	−4	−4	−12	4	6	4	2	0	−1	−3
2	−2	10	2	−10	8	5	6	5	2	−1	−2	−8
3	7	13	−12	−13	1	2	7	2	4	−1	−12	−13
4	2	9	−9	−9	−14	−6	−1	−6	−8	−1	−7	−10
5	−28	−4	18	4	−5	−9	−4	−9	4	1	16	5

TABLE VI. (*d*).—Fourier Coefficients of the Lunar Diurnal Magnetic Variation, Reduced to New Moon. The Solstitial Inequality, ½ (Summer – Winter).

No. of observatory.	West.				North.				Radial.			
	a.		*b.*		*a.*		*b.*		*a.*		*b.*	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
	(1) The 24-hour Component.											
1	67	57	−15	9	9	−8	37	49	24	11	−18	−3
2	62	57	−14	9	−3	−6	74	40	0	15	0	−4
3	60	57	14	9	−53	−5	44	30	27	18	2	−5
4	50	57	20	9	−18	−2	38	14	24	21	1	−5
5	46	57	42	9	−11	1	−20	−6	11	21	−7	−5
	(2) The 12-hour Component.											
1	78	67	−10	10	13	−9	56	58	−4	5	−3	−4
2	93	94	0	14	16	−10	69	67	8	9	−11	−8
3	120	114	47	17	−42	−9	42	59	18	14	−8	−12
4	104	129	44	19	4	−5	39	32	36	17	0	−15
5	142	132	−2	20	19	2	22	−14	5	18	−34	−16
	(3) The 8-hour Component.											
1	22	19	−4	−3	6	2	35	17	−4	1	1	−2
2	45	38	−3	−5	26	4	38	27	8	2	−10	−6
3	64	55	−14	−8	−9	4	18	29	0	4	−15	−10
4	64	71	12	−10	11	2	11	18	16	6	−4	−14
5	63	75	−26	−11	14	−1	15	−8	−1	6	−18	−15
	(4) The 6-hour Component.											
1	−4	1	6	0	−12	0	4	1	−1	0	2	0
2	−2	3	2	−1	6	1	8	2	2	0	−2	−1
3	4	6	3	−2	−8	1	0	3	0	−1	0	−2
4	6	8	2	−3	8	1	7	2	4	−1	5	−4
5	24	9	−24	−4	9	0	14	−1	−8	−1	−16	−4